

國立高雄大學九十二學年度研究所碩士班招生考試試題

系(所)別: 統計學研究所

科目: 基礎數學

滿分為100分。作答時請於答案卷上寫上題號及過程。

1. (8 %) Find the limit of

$$\lim_{\Delta x \rightarrow 0} \frac{(2+x+\Delta x)^{\frac{1}{2}} + \ln(1+(\Delta x/x))^2 - (2+x)^{\frac{1}{2}}}{\Delta x}$$

for some fixed $x > 0$.

2. (12 %) Compute $\int x e^{2x^2} \sin x^2 dx$.
3. (12 %) Suppose that function f satisfies $f(x+y) = f(x) + f(y)$ for all x, y , and that f is continuous at 0. Prove that f is continuous at every point.
4. (1) (10 %) Prove that the sequence $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \dots$ converges.
(2) (8 %) Find the limit of the above sequence.
5. Define the linear transformation $T: P_2(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$ by

$$T(f) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & 2f(0) \end{pmatrix}, \quad \text{for } f \in P_2(\mathbb{R}).$$

Here $P_2(\mathbb{R})$ denotes the collection of all polynomials of degree 2 with real coefficients and $M_{2 \times 2}(\mathbb{R})$ the set of all 2×2 matrices with entries from \mathbb{R} .

- (1) (8 %) Find a basis for the null space of T .
(2) (8 %) Find a basis for the range of T .
6. (12 %) Let A be an $n \times n$ matrix with characteristic polynomial

$$f(t) = (-1)^n t^n + a_{n-1} t^{n-1} + \dots + a_1 t + a_0.$$

Prove that A is invertible if and only if $a_0 \neq 0$.

7. (10 %) Find all $x \in \mathbb{R}$ so that

$$\text{rank} \begin{pmatrix} 3 & x & x & x \\ x & 3 & x & x \\ x & x & 3 & x \\ x & x & x & 3 \end{pmatrix} < 4.$$

8. (12 %) Let $A = \begin{pmatrix} 7 & -8 \\ 3 & -7 \end{pmatrix}$. Evaluate $\exp(A)$.