國立高雄大學九十四學年度研究所碩士班招生考試試題

系所組別：統計學研究所　　　　　　　　　　　　　　　　科目：機率論

考試時間：100 分鐘　　　　　　　　　　　　　　　　本科原始成績滿分 100 分

1. Suppose U_1 and U_2 are two independent uniform$(0,1)$. Let $R = \sqrt{-2\log U_1}$ and $\theta = 2\pi U_2$. Show that $X = R\cos\theta$ and $Y = R\sin\theta$ are two independent normal$(0,1)$. (15%)

2. Let $X_1, \cdots, X_n$ be independent Poisson random variables with common mean λ. Find the conditional distribution of X_1, given $\sum_{i=1}^n X_i$. (15%)

3. Let X_n be a chi-square distribution with degree of freedom n. Show that $(X_n - n)/\sqrt{2n}$ converges to the stadard normal in distribution as n tend to infinity. (15%)

4. Suppose Z_1, Z_2 be two independent random variables with common distribution $f(x) = \lambda\exp\{-\lambda x\}$. Let $X = Z_1$, $Y = Z_1 Z_2 + Z_2$. Find (a) $E(Y|X=x)$, (b) $E(E(Y|X))$, (c) $Var(E(Y|X))$ (d) $Var(Y|X=x)$ and (e) $E(Var(Y|X))$. (15%)

5. Let $X_1, ..., X_n$ be a random sample drawn from a population with finite variance σ^2 and $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$ where $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$. Prove that $E(S) \le \sigma$, and if $\sigma^2 > 0$, then $E(S) < \sigma$. Does S converges to σ in probability? Why? (20%)

6. Let $X_1, \cdots, X_n$ be independent exponential random variable with common mean λ^{-1}. Denote the first r $(r < n)$ order statistics of X_i as $X_{(1)} \le X_{(2)} \le \cdots \le X_{(r)}$. Define $X_{(0)} = 0$. Show that $W_j = (n-j+1)(X_{(j)} - X_{(j-1)})$, $j = 1, \cdots, r$ are also independent exponential random variable with common mean λ^{-1}. Let $T = (n-r)X_{(r)} + \sum_{i=1}^r X_{(i)}$. What is the distribution function of T? (Hint: Find the relation between T and W_j) (20%)