

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：基礎數學

考試時間：100 分鐘

本科原始成績：滿分 100 分

(1) (16%) Evaluate (a)  $\int_0^{\infty} x^3(e^{-x^2} + e^{-x})dx$ ; (b)  $\int_0^{\infty} x^{-1/2}e^{-x} dx$ .

(2) (16%) Test the following series for convergence or divergence

(a)  $\sum_{n=1}^{\infty} \frac{\log_e n}{n\sqrt{n+2}}$ ; (b)  $\sum_{k=1}^{\infty} \left(\frac{1}{k^2 + 3k + 2} + \frac{k+1}{k!}\right)$ .

(3) (10%) Find  $c$  so that

$$\lim_{x \rightarrow +\infty} \left(\frac{x+2c}{x-c}\right)^x = 8.$$

(4) (10%) Prove that the inequalities

$$1 - \frac{1}{x} < \log_e x < x - 1$$

are valid for  $x > 0, x \neq 1$ .

(5) (10%) If  $A$  and  $B$  are positive definite  $n \times n$  matrices and  $A - B$  is non-negative  $n \times n$  matrix, show that  $\det(A) \geq \det(B)$ .

(6) (10%) Let  $A$  and  $B$  be defined by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 1 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}.$$

Show that the column space of  $B$  is a subspace of the column space of  $A$ .

(7) (16%) Show that the matrix  $A$  below is positive definite, and find a matrix  $P$  such that  $P^T P = A$ .

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

(8) (12%) Let  $A$  be a  $n \times n$  idempotent matrix, that is  $A^2 = A$ . Find all possible eigenvalues of  $I - A$  where  $I$  is the  $n \times n$  identity matrix.