

國立高雄大學九十五學年度博士班招生考試試題

科目：數理統計

系所：統計學研究所

是否使用計算機：是

考試時間：100 分鐘

本科原始成績：100 分

1. Suppose that X_1, \dots, X_n be a random sample generates from an exponential distribution with a positive unknown mean λ^{-1} . Let $Y_i = I(X_i \geq M)$ ($i = 1, \dots, n$) where $M > 0$ is a known constant.
 - (i) Find the MLE $\hat{\lambda}$ of λ based on Y_i ($i = 1, \dots, n$). (10%)
 - (ii) State the asymptotic distribution of $\hat{\lambda}$ (Hint: Using delta method). (10%)
 - (iii) Create a 95% Confidence interval of λ based on $\hat{\lambda}$. (5%)
2. Let X_1, \dots, X_n be a random sample from a population with mean μ and variance σ^2 .
 - (i) Show that the estimator $\sum_{i=1}^n a_i X_i$ is an unbiased estimator of μ if $\sum_{i=1}^n a_i = 1$. (5%)
 - (ii) Find the best linear unbiased estimator of μ . That is, among all unbiased estimators of this, find the one with minimum variance. (10%)
 - (iii) Calculate the variance of the best linear unbiased estimator of μ . (5%)
3. Suppose we have two random samples (X_1, \dots, X_n) and (Y_1, \dots, Y_n) which are generated from exponential distribution with mean λ^{-1} and τ^{-1} respectively. Let
$$T = \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n X_i + \sum_{i=1}^n Y_i}.$$
 - (i) Find the distribution of T (Hint: The relationship among exponential, gamma, chi-square and F- distributions). (15%)
 - (ii) Give the likelihood ratio test statistic L_{RT} for testing $H_0 : \lambda = \tau$ vs $H_a : \lambda \neq \tau$ in term of T . (10%)
4. The random variables X_1, \dots, X_n are independent and X_i is $N(\theta_i, 1)$.
 - (i) Show that the most powerful size 0.05 test of the null hypothesis that each θ_i is zero against the alternative that $\theta_i = 0.5$ for $i = 1, \dots, r$ and $\theta_i = -0.5$ for $i = r + 1, \dots, n$ has critical region $\{(X_1, \dots, X_n) | \sum_{i=1}^r X_i - \sum_{i=r+1}^n X_i > 1.645\sqrt{n}\}$. (15%)
 - (ii) How large must n be to ensure that the power of this test is at least 0.9? (15%)