國立高雄大學九十六學年度研究所碩士班招生考試試題

科目：機率論　　系所： 統計學研究所　　是否使用計算機：是
考試時間：100 分鐘　　本科原始成績：100 分

1. Prove each of the following statements. (Assume that any conditioning event has positive probability)

   (1) If $P(B)=1$, then $P(A|B)=P(A)$ for any A. (5%)

   (2) Assume that $P(A)>0$ and $P(B)>0$. If A and B are independent, then they cannot be mutually exclusive, and if A and B are mutually exclusive, then they cannot be independent. (5%)

2. Prove that the following functions are cdfs.
   (1) $1-\exp(-x), x\in(0,\infty)$, (2) $\exp(-e^{-x}), x\in(-\infty,\infty)$. (10%)

3. Consider the following two pdfs,

$$f_1(x)=\frac{1}{\sqrt{2\pi}x}\exp(-(\log x)^2/2), x\geq 0, \text{ and } f_2(x)=f_1(x)[1+\sin(2\pi\log x)], x\geq 0.$$

   Show that

   (1) If $X_1\sim f_1(x)$, then $E(X_1^r)=\exp(r^2/2), r=0,1,2,\ldots$. (10%)

   (2) Suppose $X_2\sim f_2(x)$. Then $E(X_1^r)=E(X_2^r)$ for $r=0,1,2,\ldots$. (10%)

4. Prove the following inequalities.

   (1) Let X be a random variable with moment-generating function, $M_X(t), -h<t<h$. Show that

$$P(X\geq a)\leq e^{-at}M_X(t), 0<t<h, \text{ and } P(X\leq a)\leq e^{-at}M_X(t), -h<t<0.$$

   (10%)

   (2) Let $X_1,\ldots,X_n$ be iid with mgf $M_X(t), -h<t<h$, and let $S_n=\sum_{i=1}^n X_i$. Show that

$$P(S_n>a)\leq e^{-at}[M_X(t)]^n, 0<t<h, \text{ and } P(S_n\leq a)\leq e^{-at}[M_X(t)]^n, -h<t<0.$$

   (10%)

背面尚有試題

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5. Suppose U_1 and U_2 are iid Uniform(0,1). Let

$$X_1 = \cos(2\pi U_1)\sqrt{-2\log U_2} \text{ and } X_2 = \sin(2\pi U_1)\sqrt{-2\log U_2}.$$

Prove that X_1 and X_2 are independent N(0,1) random variables. (10%)

6. Let X and Y be independent N(0,1) random variables. Define a new random variable, Z, by

$$Z = \begin{cases} X & \text{, if } XY > 0 \\ -X & \text{, if } XY < 0 \end{cases} .$$

Show that Z has a normal distribution. (10%)

7. Find the pdf of $\prod_{i=1}^{n} X_i$, where X_is are independent Uniform(0,1) random variables. (10%)

8. Let $X \sim \text{Poisson}(\theta)$ and $Y \sim \text{Poisson}(\lambda)$, and X and Y are independent. Find the distribution of $X|X+Y$. (10%)