

國立高雄大學九十六學年度研究所碩士班招生考試試題

科目：基礎數學
考試時間：100 分鐘

系所：統計學研究所
本科原始成績：100 分

是否使用計算機：是

1. (8%) Find the limit (if it exists)

$$\lim_{n \rightarrow \infty} (1 + a^2)(1 + a^4) \dots (1 + a^{2^n})$$

2. (12%) Find the limit

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n(n+1)(n+2)}$$

3. (10%) Show that the equation $5ax^4 + 2bx = a + b$ has at least one zero in $(0, 1)$.

4. Evaluate the following integrals

(a) (8%) $\int_0^{\pi/2} \frac{\tan^3 x}{\tan^3 x + \cot^3 x} dx$

(b) (12%) $\int_0^8 \int_{\sqrt[3]{y}}^2 \frac{y}{\sqrt{16+x^7}} dx dy$

5. Let $\mathcal{C}^\infty(\mathbb{R})$ be the space of infinitely differentiable functions on \mathbb{R} and $T : \mathcal{C}^\infty(\mathbb{R}) \rightarrow \mathcal{C}^\infty(\mathbb{R})$ be defined by $T(f) = f + f'$. Denote $\mathbf{P}_2(\mathbb{R}) = \{a + bx + cx^2 : a, b, c, \in \mathbb{R}\}$.

(a) (5%) Find the null space $\mathbf{N}(T)$ of T . Is T invertible ?

(b) (5%) Show that $T(\mathbf{P}_2(\mathbb{R})) \subseteq \mathbf{P}_2(\mathbb{R})$, i.e. $\mathbf{P}_2(\mathbb{R})$ is T -invariant.

(c) (5%) Let S be the restriction of T on $\mathbf{P}_2(\mathbb{R})$. Find $[S]_\beta$, the matrix representation of S with respect to the ordered basis $\beta = \{1, x, x^2\}$.

(d) (10%) Is S is invertible? If your answer is "yes", find $[S]_\beta^{-1}$.

6. For each of the following matrices $A \in M_{n \times n}$, test A for diagonalizability and then find e^A

(a)(10%) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b)(15%) $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{bmatrix}$.