國立高雄大學九十六學年度研究所碩士班招生考試試題

科目：數理統計　　系所： 統計學研究所　　是否使用計算機：是
考試時間：100 分鐘　　本科原始成績：100 分

(1) Suppose $X = (X_1, \cdots, X_n)$ be a random sample with p.d.f $f(x) = \lambda \exp\{-\lambda x\}$ where $x \geq 0, \lambda > 0$.

(a) Show that $T(X) = \sum_{i=1}^{n} X_i$ is a complete sufficient statistic of λ ; (5%)

(b) Find an UMVUE of $\theta = \exp\{-\lambda\}$; (10%)

(c) Use the Delta Method to derive a 95% confidence interval of θ. (10%)

(2) Let $X_1, \cdots, X_n$ be i.i.d Poisson(λ).

(a) Find the UMP, size α, test of $H_0 : \lambda \leq \lambda_0$ vs $H_a : \lambda > \lambda_0$; (10%)

(b) Show that there does not exist a UMP, size α, test of $H_0 : \lambda = \lambda_0$ vs $H_a : \lambda \neq \lambda_0$; (10%)

(c) Consider the specific case $H_0 : \lambda \leq 1$ vs $H_a : \lambda > 1$, use the Central Limit Theorem to determine the sample size n so that the UMP test satisfies

$P(\text{reject } H_0 \mid \lambda = 1) = 0.05$ and $P(\text{reject } H_0 \mid \lambda = 2) = 0.95$. (Recall: $\Phi(1.64) = 0.95$) (10%)

(3) Let $X_1, \cdots, X_n$ be a random sample from the uniform distribution on the interval $[0, \theta]$.

(a) Find a minimal sufficient statistic of θ; (5%)

(b) Find an UMVUE of θ; (5%)

(c) Compare the variance of the UMVUE with the Cramer-Rao lower bound for the variance of an unbiased estimator and explain why this lower bound is not applicable in this instance. (5%)

(4) Let X and Y be i.i.d N(0,1) random variables, and define $Z = \min(X, Y)$. Prove that Z^2 has chi-square distribution with degree of freedom 1. (15%)

(5) Suppose that one has n random pairs of measurements (X_i, Y_i) $(i = 1, \cdots, n)$ with joint p.d.f.

$f_{X_i, Y_i}(x_i, y_i) = \lambda_i \tau_i \exp\{-\lambda_i x_i - \tau_i y_i\}$ where $\lambda_i, \tau_i > 0$ $(i = 1, \cdots, n)$ are $2n$ unknown positive constants. Assume that the n pair unknown constants (λ_i, τ_i) $(i = 1, \cdots, n)$ lie on a straight line which pass through the origin. It is required to estimate the slope of that straight line. Obtain a maximum likelihood solution of this problem, and elaborate the computational details. (15%)