國立高雄大學九十六學年度博士班招生考試試題

科目：數理統計　　系所：統計學研究所　　是否使用計算機：是
考試時間：100 分鐘　　本科原始成績：100 分

1. Markov's Inequality is known as

$$P(|X| \geq \varepsilon) \leq E(|X|)/\varepsilon$$

for an arbitrary random variable X and $\varepsilon > 0$.

(a) For every fixed $\varepsilon > 1$, find a distribution for X with $E(X) = 0$ that gives equality in Markov's inequality. (10 points)

(b) Prove for an arbitrary random variable X,

$$P(|X| \geq \varepsilon) \leq \frac{E(\cosh(X)) - 1}{\cosh(\varepsilon) - 1}.$$

Here $\cosh(x) = (e^x + e^{-x})/2$. (10 points)

2. Let $X_1, \ldots, X_n$ be a sample from a population with the density

$$f(x) = \theta(\theta+1)x^{\theta-1}(1-x),$$

$0 < x < 1, \theta > 0$. Find a method of moments estimate of θ. (10 points)

3. Let $\mathbf{X} = (X_1, \ldots, X_n)$ be a sample from a $U(0, \theta)$ population, where $0 < \theta < \infty$ and θ is unknown. Then find the U.M.V.U. estimate of θ. (15 points)

4. Suppose that $X_1, \ldots, X_n$ is a sample from the pdf, $f(x|\theta) = \exp(-(x-\theta))$, $x > \theta$ and $\theta \in \Re$. Then construct the U.M.P. level α test for

$$H_0 : \theta \leq \theta_0 \text{ vs. } H_1 : \theta > \theta_0.$$

(15 points)

5. Let $X_1, \ldots, X_n$ be a sample from a $N(\mu, \sigma^2)$ population. Here σ^2 is assumed to be known. Suppose $z(p)$ denote the pth quantile of the standard normal distribution.

(a) Show $[\bar{X} - \sigma z(1-\alpha_1)/\sqrt{n}, \bar{X} + \sigma z(1-\alpha_2)/\sqrt{n}]$ is a level $(1-\alpha)$ confidence interval for μ, where $\bar{X} = \sum_{i=1}^n X_i/n$ and $\alpha_1 + \alpha_2 = \alpha$. (10 points)

(b) From (a), the shortest level $(1-\alpha)$ interval is obtained by taking $\alpha_1 = \alpha_2 = \alpha/2$. (10 points)

6. Let $X_1, \cdots, X_n$ be a sample from $N(\theta, \sigma^2)$ with σ^2 known. For a fixed number a, let $p = P(X_i > a)$.

(a) Find the maximum likelihood estimate of p, $\hat{p}$. (10 pints)

(b) Find the asymptotic distribution of $\sqrt{n}(\hat{p} - p)$. (10 points)