國立高雄大學九十七學年度博士班招生考試試題

科目：機率論　　系所： 統計學研究所　　是否使用計算機：否
考試時間：100 分鐘　　本科原始成績：100 分

1. Suppose that $X_1, \cdots, X_{2n}$ is a random sample with variance σ^2. Let $U = \frac{1}{n}\sum_{i=1}^{2n} X_{2i-1}$ and $V = \frac{1}{2n}\sum_{i=1}^{2n} X_i$. Consider Chebyshev inequality and Center Limit Theorem, respectively, to suggest a value of n which such that $P(|U - V| \le \sigma) \ge 0.95$. Which method gives a smaller suggested value of n? Why? (40%)

2. Let (X, Y) be a random vector with joint density function $f(x, y) = k|xy|, 0 \le x^2 + y^2 \le 1$. (a) Find the value of k. (b) Find $E(X^2 e^{Y+Y^{-1}} | Y = y)$. (25%)

3. Find the value of $Cov(2X, E(Y | X))$ in terms of $Cov(X, Y)$. (15%)

4. Let $X_1, \cdots, X_n$ be i.i.d. random variables which has distribution $N(\mu, \sigma^2)$. Prove that the sample mean $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ are independent to each other. (20%)