

國立高雄大學九十七學年度博士班招生考試試題

科目：數理統計
 考試時間：100 分鐘

系所：統計學研究所
 本科原始成績：100 分

是否使用計算機：是

1. Let X_1, X_2, X_3 be independent Poisson random variables with mean $\lambda_1, \lambda_2, \lambda_3$, respectively, and create the random variables

$$Y_1 = X_1 + X_3 \quad \text{and} \quad Y_2 = X_2 + X_3.$$

- (a) Show that $\text{Cov}(Y_1, Y_2) = \lambda_3$. (5%)

- (b) Define $Z_i = I(Y_i = 0)$ and $p_i = e^{-(\lambda_i + \lambda_3)}$. Show that Z_i are Bernoulli (p_i) with

$$\text{Corr}(Z_1, Z_2) = \frac{p_1 p_2 (e^{\lambda_3} - 1)}{\sqrt{p_1(1-p_1)} \sqrt{p_2(1-p_2)}}. \quad (10\%)$$

- (c) Show that the correlation of Z_1 and Z_2 is not unrestricted in the range $[-1, 1]$, but

$$\text{Corr}(Z_1, Z_2) \leq \min \left\{ \sqrt{\frac{p_2(1-p_1)}{p_1(1-p_2)}}, \sqrt{\frac{p_1(1-p_2)}{p_2(1-p_1)}} \right\}. \quad (10\%)$$

2. Let X_1, \dots, X_n be independent with pdfs $f_{X_i}(x | \theta) = e^{i\theta - x} I_{[i\theta, \infty)}(x)$. Prove that

$T = \min_i (X_i / i)$ is a sufficient statistic for θ . Based on T , find the $1 - \alpha$ confidence interval for θ of the form $[T + a, T + b]$ which is of minimum length. (20%)

3. Complete the following problems:

- (a) If χ_{2Y}^2 is a chi squared random variable with $Y \sim \text{Poisson}(\lambda)$, show that $E(\chi_{2Y}^2) = 2\lambda$,

$\text{Var}(\chi_{2Y}^2) = 8\lambda$, the mgf of χ_Y^2 is given by $\exp(-\lambda + \frac{\lambda}{1-2t})$, and

$$\frac{\chi_{2Y}^2 - 2\lambda}{\sqrt{8\lambda}} \rightarrow n(0, 1)$$

as $\lambda \rightarrow \infty$. (Use moment generating functions.) (15%)

- (b) Now evaluate

$$P\left(\frac{nb}{nb+1} \chi_{2(Y+a), 1-\alpha/2}^2 \leq \chi_{2Y}^2 \leq \frac{nb}{nb+1} \chi_{2(Y+a), \alpha/2}^2\right)$$

as $\lambda \rightarrow \infty$ by first standardizing χ_{2Y}^2 . Show that the standardized upper limit goes to $-\infty$ as $\lambda \rightarrow \infty$, and hence the coverage probability goes to 0. (15%)

4. Let X_1, \dots, X_n be a random sample from a $n(\theta, \sigma^2)$ population. Consider testing $H_0: \theta \leq \theta_0$ versus $H_1: \theta > \theta_0$.

國立高雄大學九十七學年度博士班招生考試試題

科目：數理統計
考試時間：100 分鐘

系所：統計學研究所
本科原始成績：100 分

是否使用計算機：是

- (a) If σ^2 is known, show that the test that rejects H_0 when

$$\bar{X} > \theta_0 + z_\alpha \sqrt{\sigma^2 / n}$$

is a test of size α . Show that the test can be derived as an LRT. (10%)

- (b) Show that the test in part (a) is a UMP test. (5%)

- (c) If σ^2 is unknown, show that the test that rejects H_0 when

$$\bar{X} > \theta_0 + t_{n-1, \alpha} \sqrt{S^2 / n}$$

is a test of size α . Show that the test can be derived as an LRT. (10%)