

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：機率論
考試時間：100 分鐘

系所：
統計學研究所碩士班
本科原始成績：100 分

是否使用計算機：是

1. Suppose we have a population of r distinct objects labeled $1, 2, \dots, r$. Objects are drawn with replacement until exactly $k \leq r$ distinct objects have been obtained. Let S_k denote the size of the sample required. Compute ES_k and $VarS_k$. (15%)

2. Consider the problem of matching n objects, and let i and r denote distinct specified positions.

(a) What is the probability that a match occurs at position i and no match occurs at position r ?

(b) Given that there is no match at position r what is the probability of a match in position i ? (10%)

3. Let Y have a distribution function given by

$$F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

Find a transformation $G(U)$ such that, if U has a uniform distribution on the interval $(0,1)$, $G(U)$ has the same distribution as Y . (10%)

4. Let X_1, \dots, X_n be iid with pdf

$$f_X(x) = \begin{cases} \frac{a}{\theta^a} x^{a-1}, & \text{if } 0 < x < \theta. \\ 0, & \text{otherwise} \end{cases}$$

Let $X_{(1)} < \dots < X_{(n)}$ be the order statistics. Show that $X_{(1)}/X_{(2)}, X_{(2)}/X_{(3)}, \dots, X_{(n-1)}/X_{(n)}$,

and $X_{(n)}$ are mutually independent random variables. Find the distribution of each of them.

(15%)

5. Let X be a ransom variable having density f given by

$$f(x) = \begin{cases} 1/18, & x = 1, 3, \\ 16/18, & x = 2. \end{cases}$$

Show that there is a value of δ such that $P(|X - EX| \geq \delta) = VarX / \delta^2$, so that in general the bound given by Chebyshev inequality cannot be improved. (5%)

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6. Let X_1, X_2, \dots, X_n be iid with moment generating function $M_X(t)$, $-h < t < h$, and let

$$S_n = \sum_{i=1}^n X_i \quad \text{and} \quad \bar{X}_n = S_n/n.$$

(a) Show that $P(S_n > a) \leq e^{-at} [M_X(t)]^n$, for $0 < t < h$, and $P(S_n \leq a) \leq e^{-at} [M_X(t)]^n$, for $-h < t < 0$.

(b) Use the facts that $M_X(0) = 1$ and $M'_X(0) = EX$ to show that, if $EX < 0$, then there is a $0 < c < 1$ with $P(S_n > a) \leq c^n$. Establish a similar bound for $P(S_n \leq a)$.

(15%)

7. Let X_1, X_2, \dots, X_n be independent χ^2 -distributed random variables, each with 1 degree of freedom. Define Y as

$$Y = \sum_{i=1}^n X_i.$$

Therefore, Y has a χ^2 distribution with n degrees of freedom.

(a) Use the preceding representation of Y as the sum of the X s to show that

$$Z = (Y - n)/\sqrt{2n} \quad \text{has an asymptotic standard normal distribution.}$$

(b) A machine in a heavy-equipment factory produces steel rods of length Y , where Y is a normally distributed random variable with mean 6 inches and variance 0.2. The cost C of repairing a rod that is not exactly 6 inches in length is proportional to the square of the error and is given, in dollars, by $C = 4(Y - \mu)^2$. If 50 rods with independent lengths are produced in a given day, approximate the probability that the total cost for repairs for that day exceeds \$48. (Express the answer in terms of Φ)

(15%)

8. Suppose that Y_1 and Y_2 are independent exponentially distributed random variables, both with mean β , and define $U_1 = Y_1 + Y_2$ and $U_2 = Y_1/Y_2$.

(a) Find the joint density function of (U_1, U_2) .

(b) Are U_1 and U_2 independent? Why? (15%)