Baseline Survival Function Estimators under Proportional Hazards Assumption

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在比例風險假設下的存活函數估計

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摘要

Cox (1972) 提出用部分概似估計法來估計無特定基線風險函數之比例風險迴歸模型的風險係數。當估計出風險係數 β 後，我們有時亦有興趣估計其基線存活函數。因此，Breslow (1972) 及 Kalbfleisch & Prentice (1973) 提供了兩個不同的建構基線存活函數之估計量。然而，這兩個估計量都有些缺點。Breslow 的估計量可能會是不合理的負值，而當有重複死亡時間 (tie) 出現時，Kalbfleisch & Prentice 的估計量並無公式解。在這篇文章中，如同 Breslow 及 Kalbfleisch & Prentice，我們提出了另一個亦是基於無母數最大概似估計 (NPMLE) 法的估計量，然而，此估計量在所有情形下都有公式解且不會是負值。我們亦做了一系列的模擬來觀察在特定的模型及有限樣本下，此三估計量的比較。

關鍵字 : 比例存活迴歸模型, 基線風險函數, 基線存活函數, 無母數最大概似, 最大概似估計量.

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Abstract

Cox (1972) proposed the partial likelihood technique to estimate the risk coefficients of proportional hazards regression model without specify the baseline hazards function. Once the risk coefficients β are estimated, we may interest in estimating the corresponding baseline survival function. Therefore, Breslow (1972) and Kalbfleisch & Prentice (1973) provided two different procedures as the baseline survival function estimators. However, these two estimators have some drawbacks. Breslow's estimator may give negative value, and Kalbfleisch & Prentice's estimator has no close form when ties data is presented. In this work, we propose another estimator which is also based on NPMLE technique as Breslow's and Kalbfleisch & Prentice's estimators, however, it has a close form in all cases and never give negative value. To compare these three estimators, we run a series of simulations to see the finite sample performance of them under some specified models.

Keywords : Proportional hazards regression model, Baseline hazards function, Baseline survival function, nonparametric maximum likelihood, MLE.

1 Introduction

The Cox (1972) proportional hazard model is a useful technique in investigating the effects of diagnostic variables associated with a system on its life period. It has a semi-parametric form, specifically,

$$h(t|z) = h_0(t)e^{\beta \mathbf{z}},$$

where $h_0(t)$ is the baseline hazard function, z is a vector of possibly time-dependent covariables, and β is a vector of unknown parameters. It is well known that the efficient inference can be made based on a partial likelihood approach without specified $h_0(t)$. In addition, the proportional hazard model assumption provides a reasonable empirical approximation to the association between covariates and censored failure times in applications.

Although the partial likelihood estimator has no close form, it can be obtained through Newton-Raphson method easily when there are no ties among exact failure time in the recorded data . If ties are presented, the true partial likelihood function involves permutations and the computation can be time-consuming. In this case, several approximated partial likelihood functions have been proposed by Breslow (1974), Efron (1977), and Kalbfleisch & Prentice (1973) for handling ties in the Cox proportional hazard model. Besides, partial likelihood has also been used by Peto & Peto (1972) for constructing asymptotically efficient rank test statistics in the two-sample problem with censored survival data. Efron (1977) and Oakes (1977) developed asymptotic efficiency formulae for maximum partial likelihood estimators within Cox's (1972) regression model, while Tsiatis (1981) was the first to prove consistency of such estimators.

After obtaining the estimators $\widehat{\beta}$, we may interest in estimating the baseline survival function. In the discussion followed by Cox's paper, all these estimators reduce to the product limit estimate (Kaplan and Meier, 1958) when there are no covariates. Specifically, Oakes (1972) suggested a step-function estimate instead of the point-wise estimate for $h_0(t)$. Kalbfleisch & Prentice (1973) proposed a step-function estimate for $h_0(t)$ where the baseline hazard function $h_0(t)$ is assumed to be a constant between convenient (but arbitrary) subdivisions of the time scale. Breslow (1972) gave a similar estimate for $h_0(t)$ but he defined subdivisions of the time scale as those points at which death occurs.

Due to the appliance of the nonparametric maximum likelihood estimation (NPMLE) technique, Breslow's (1972) and Kalbfleisch & Prentice's (1973) estimators are generally used in real case studies. However, they have some drawbacks. Breslow's estimator of the baseline survival function may give a negative value. Kalbfleisch & Prentice's estimator of the baseline survival function $S_0(t)$ has a close form, when no tie, otherwise, an iterative solution is required. In this thesis, we propose another estimator which is also create through NPMLE technique. However, this estimator has a close form in all cases as Breslow's estimator and disposed of the drawback of Breslow's estimator.

In the next section, we outline the partial likelihood technique with Cox model. Then, the baseline survival function and baseline cumulative hazard function estimators given by Breslow (1972) and Kalbfleisch & Prentice (1973) are reviewed in section 3.1 and 3.2 respectively. In section 3.3, we propose another estimator through an approximate likelihood function other than theirs. In section 4, a series of simulation studies are shown to compare the efficiency of these three estimators. Last, the conclusion is provided in section 5.

2 Proportional Hazards Model

Let T be a nonnegative variable representing the failure time of an individual in the population. The distribution of failure time, T, can be represented in the usual manner in terms of density or distribution functions as well as in more specialized ways such as the hazard function. Specifically, the hazard function at time t among individuals with covariate z is defined as

$$h(t|z) = \lim_{\Delta t \to 0} P(t \leq T < t + \Delta t | T \geq t, z)/\Delta t,$$

which represents the "risk" of failure at any time t, given that individual has not failed prior to t. Since the notion of hazard rate is basic and conceptually simple, $h(t|z)$ provides a convenient starting point for modeling the relationship of hazard functions among different covariates z.

One such model, introduced by Cox (1972), provides the basis for many of the discussions. This model assumes that covariates affect the hazard function in a multiplicative manner based to

$$h(t|z) = h_0(t)e^{\beta z},$$

where β is a row vector of p unknown parameters and $h_0(t)$ is an arbitrary baseline hazard function. The factor $e^{\beta z}$ describes the risk of failure for an individual with regression variable z related to factor $e^{\beta z}$ at a standard value $z = 0$. Since the ratio of hazard functions corresponding to any two z-values is not dependent on t, Cox model is often referred to as the proportional hazards model. In fact the Cox model can be extended to include time dependent covariates so that it is not anymore proportional hazards. Without loss of generality, in this manner, we focus on the cases of time independent covariates.

2.1 The Full Likelihood Function

We consider survival studies in which n individuals are put on test and data of the form $(t_i, \delta_i, z_i)(i = 1, \cdots, n)$, are collected. Here t_i is the minimum of the exact failure time T_i and the censoring time C_i of the ith individual, $\delta_i = I(T_i \leq C_i)$ is an indicator variable represents the failure status, and z_i is the corresponding covariate that may be a vector-value. In addition, the survival function of the ith individual is $S(t|z_i) = P(T_i > t|z_i)$. The corresponding density function is $f(t|z_i)$, where T_i is the exact failure time. For the time being, we restrict our attention to absolutely continuous failure times T_i. Furthermore, we assume that the censoring time C_i of the ith individual is a random variable with survival and density functions $G(t|z_i)$ and $g(t|z_i)$ $(i = 1, \cdots, n)$ respectively and that given $z_1, \ldots, z_n$, the $C_1, \ldots, C_n$ are stochastically independent of each other and of the independent failure times $T_1, \ldots, T_n$. Therefore, the full likelihood function of the data (t_i, δ_i, z_i), conditional on $z_1, \ldots, z_n$, is

$$L(\beta|\underset{\sim}{t}, \underset{\sim}{\delta}, \underset{\sim}{z})) = \prod_{i=1}^{n}[f(t_i|z_i)G(t_i|z_i)]^{\delta_i}[S(t_i|z_i)g(t_i|z_i)]^{1-\delta_i}.$$

Since the censoring time is noninformative, the full likelihood function can be rewritten as

$$\begin{aligned} L(\beta|\underset{\sim}{t}, \underset{\sim}{\delta}, \underset{\sim}{z})) &\propto \prod_{i=1}^{n}[f(t_i|z_i)]^{\delta_i}[S(t_i|z_i)]^{1-\delta_i} && (1) \\ &= \prod_{i=1}^{n}[h(t_i|z_i)]^{\delta_i}S(t_i|z_i). && (2) \end{aligned}$$

Replacing $h(t|z)$ by $h_0(t)\exp\{\beta z\}$, the full likelihood function becomes

$$L(\beta|\underset{\sim}{t}, \underset{\sim}{\delta}, \underset{\sim}{z})) = \prod_{i=1}^{n}[h_0(t_i)\exp\{\beta z_i\}]^{\delta_i}\exp[-\int_0^{t_i} h_0(u)\exp\{\beta z_i\}du]. \qquad (3)$$

As we can see, this full likelihood function contains an unspecified baseline hazard function so that the estimate of β is difficult to obtain. In the next section, we will review the partial likelihood technique for estimate β.

2.2 The Partial Likelihood Function

A remarkable feature of the proportional hazards model is that it admits feasible estimation of the relative risk parameters, β, without specifying the baseline hazard function $h_0(t)$.

Suppose that there are no tie among the failure times and let $t_{(1)} < t_{(2)} < \cdots < t_{(k)}$ be k distinct failure times in the observed data. Let $z_{(i)}$ denote the covariate vector corresponding to $t_{(i)}$ and let $R(t_{(i)})$ be the set of all individuals who are still under study at the time just prior to $t_{(i)}$.

The primary method of analysis is called partial likelihood. It formed the basis of the Cox (1972) analysis of proportional hazards model , and was discussed further in Cox (1975). If the hazard function given z, $h(t|z)$ is specified up to a finite number of parameters, ordinary parametric likelihood method can be applied to

$$L(\beta) = \prod_{i=1}^{k} \frac{\exp\{\beta z_{(i)}\}}{\sum_{j \in R_{(t_{(i)})}} \exp(\beta z_j)}.$$

It should be noted that the presence of censoring, except in a few extremely special cases, precludes the possibility of tractable exact distribution theory. For example, standard asymptotic likelihood methods would ascribe an asymptotic normal distribution to the maximum partial likelihood estimator $\hat{\beta}$, with mean β and variance estimator $I^{-1}(\hat{\beta})$, where

$$I(\beta) = -\frac{\partial^2 l(\beta)}{\partial \beta^2} = \sum_{i=1}^{k} \left[\frac{\sum_{j \in R_{(t_{(i)})}} z_j^2 \exp(\beta z_j)}{\sum_{j \in R_{(t_{(i)})}} \exp(\beta z_j)} - \left(\frac{\sum_{j \in R_{(t_{(i)})}} z_j \exp(\beta z_j)}{\sum_{j \in R_{(t_{(i)})}} \exp(\beta z_j)} \right)^2 \right],$$

is the Fisher information. A Newton-Raphson iteration is often a convenient procedure for computing of $\hat{\beta}$.

3 Baseline Survival Function Estimators

According to the full likelihood function that the aspect narrates, Breslow (1972) provided an estimate for $\hat{h}_0(t)$, which is obtained by maximizing of $h_0(t)$ in which the parameters β are substituted by the maximum partial likelihood estimators $\hat{\beta}$. The estimator of the baseline survival function $S_0(t)$ is given by

$$\hat{S}_{0,B}(t) = \prod_{i|t_{(i)}<t} \left(1 - \frac{d_i}{\sum_{j\in R(t_{(i)})} \exp\{\beta z_j\}}\right),$$

where d_i is the number of death at time $t_{(i)}$ and $R(t_{(i)})$ is the risk set at time t_i. Clearly, $\hat{S}_{0,B}(t)$ might give a negative value if $d_i > \sum_{j\in R(t_{(i)})} \exp\{\beta z_j\}$.

Later, Kalbfleisch & Prentice (1973) proposed another estimator under Lehmann alternative model. This method is based on a nonparametric full likelihood construction that produces the generalized MLE for $\hat{S}_0(t)$. If there are no ties, their proposed estimator of baseline survival function is $\hat{S}_{0,KP}(t) = \prod_{t_{(i)}<t} \hat{\alpha}_i$

$$\hat{\alpha}_i = \left[1 - \frac{\exp\{\beta z_{(i)}\}}{\sum\limits_{j\in R_i} \exp\{\beta z_j\}}\right]^{\exp\{-\beta z_{(i)}\}}.$$

Otherwise, we have to estimate α_i numerically. Newton's method is one of the useful numerical methods which can be applied in this case.

In sections 3.1 and 3.2, we review Breslow's and Kalbfleisch & Prentice estimators, respectively, in the case with no tie. The arguments can be easily extended to data with ties. In section 3.3, we propose another estimator of baseline survival function which starts by a similar argument as Kalbfleisch & Prentice, however, it has a similar close form as Breslow's estimator. In section 4, we give a modification to Breslow's and ours proposed estimators, which shows to have better efficiency in our simulation studies.

3.1 Breslow's Estimator

To obtain the baseline hazards function, Breslow (1972) starts by the full likelihood function (3) with β replace by $\hat{\beta}$. As we can see, there are two main components about $h_0(t)$ in the likelihood function (3). The second component,

$\exp\{-\int_0^{t_i} h_0(u)du\}$, tells us that to maximize (3), we should assign as small value to $h_0(t)$ as possible. However, the first component, $[h_0(t_i)]^{\delta_i}$, tells us that the larger value of $h_0(t_i)$ when $\delta_i = 1$ will give a larger value in the likelihood function (3). This leads us to give $\hat{h}_0(t) = 0$ for all $t \notin \{t_{(1)}, \cdots, t_{(k)}\}$. Thus, $\{\hat{h}_0(t_{(1)}), \cdots, \hat{h}_0(t_{(k)})\}$ that maximize (3) will also maximize

$$\ell(h_0(t_{(i)})) = \sum_{i=1}^{k} [\ln h_0(t_{(i)}) + \hat{\beta} z_{(i)}] - \sum_{i=1}^{k} h_0(t_{(i)}) \sum_{j \in R(t_{(i)})} \exp\{\hat{\beta} z_j\}. \tag{4}$$

Differentiating (4) with respect to $h_0(t_{(i)})$ gives the maximum likelihood estimate of $h_0(t_{(i)})$ as a solution to

$$\hat{h}_0(t_{(i)}) = \frac{1}{\sum_{j \in R(t_{(i)})} \exp(\hat{\beta} z_j)}.$$

Then, the baseline survival function estimate is given by

$$\hat{S}_{0,B}(t) = \prod_{i | t_{(i)} < t} \left[1 - \frac{1}{\sum_{j \in R(t_{(i)})} \exp\{\hat{\beta} z_j\}} \right]. \tag{5}$$

It is important to note that the baseline survival function estimate $\hat{S}_{0,B}(t)$ may give a negative value. When $\hat{S}_{0,B}(t) < 1$, we revise $\hat{S}_{0,B}(t)$ as 0.

Although Breslow did not discretize the baseline survival function at the beginning, the suggested estimator of $S_0(t)$ is still a step function with jump points $\{t_{(1)}, \cdots, t_{(k)}\}$. In addition, Breslow's estimator of baseline survival function is a semi-parametric profile likelihood estimator. The hazard function $h_0(t|z)$ is assumed proportional to the baseline hazard function $h_0(t)$. The vector parameters β are replaced by partial maximum likelihood estimates $\hat{\beta}$. But the baseline hazard function $h_0(t)$ is unspecified when estimate.

3.2 Kalbfleisch & Prentice Estimator

Assume that baseline survival function $S_0(t)$ has only jump points on the k distinct failure times $t_{(1)}, \ldots, t_{(k)}$. That is, we use a discrete function to approach a continuous function. Then, replacing $S(t|z)$ by $S_0(t)^{\hat{\beta} z}$ to the full likelihood function

(1), we have

$$
\begin{aligned}
& L(S_0(t_{(1)}), \cdots, S_0(t_{(k)})|\underset{\sim}{t}, \underset{\sim}{z}) \\
= & \prod_{i=1}^{n} [S(t_i|z_j) - S(t_i + \varepsilon|z_i)]^{\delta_i} [S(t_i + \varepsilon|z_i)]^{1-\delta_i} \\
= & \prod_{i=1}^{n} \left[S_0(t_i)^{\exp\{\hat{\beta} z_i\}} - S_0(t_i + \varepsilon)^{\exp\{\hat{\beta} z_i\}}\right]^{\delta_i} \left[S_0(t_i + \varepsilon)^{\exp\{\hat{\beta} z_i\}}\right]^{1-\delta_i} \\
= & \prod_{1 \le i \le k} \prod_{j \in D_i} \left(S_0(t_{(i)})^{\exp\{\hat{\beta} z_j\}} - S_0(t_{(i+1)})^{\exp\{\hat{\beta} z_j\}}\right) \prod_{j \in C_i} S_0(t_{(i+1)})^{\exp\{\hat{\beta} z_j\}}.
\end{aligned}
$$

Here, D_i is the set of individuals failing at $t_{(i)}$ and C_i is the set of individuals censored in $[t_{(i)}, t_{(i+1)}), i = 0, \cdots, k$. In addition, $t_{(0)} = 0$, $t_{(k+1)} = \infty$ and D_0 is empty set.

It is noteworthy that the Cox model can also be written as $S(t|z) = S_0(t)^{\exp\{\hat{\beta} z\}}$ in the continuous case. However, it is not true in the discrete case. In fact, $S(t|z) = S_0(t)^{\exp\{\hat{\beta} z\}}$ is called Lehmann alternative model.

Let $\alpha_i = P(T > t_{(i)}|T \ge t_{(i)}, z = 0)$ denote the conditional survival probability at time $t_{(i)}$ for a baseline subject. Therefore, we have $S_0(t) = \prod_{t_{(i)} < t} \alpha_i$. This leads to the following likelihood function

$$
\begin{aligned}
L = & \prod_{1 \le i \le k} \left[\prod_{j \in D_i} \left[\left(\prod_{t_{(l)} < t_{(i)}} \alpha_l\right)^{\exp\{\hat{\beta} z_j\}} - \left(\prod_{t_{(l)} \le t_{(i)}} \alpha_l\right)^{\exp\{\hat{\beta} z_j\}}\right]\right] \left[\prod_{j \in C_i} \left(\prod_{t_{(l)} \le t_{(i)}} \alpha_l\right)^{\exp\{\hat{\beta} z_j\}}\right] \\
= & \prod_{1 \le i \le k} \left[\prod_{j \in D_i} \left[\left(\prod_{t_{(l)} < t_{(i)}} \alpha_l\right)^{\exp\{\hat{\beta} z_j\}} \left(1 - \alpha_i^{\exp\{\hat{\beta} z_j\}}\right)\right]\right] \left[\prod_{j \in C_i} \left(\prod_{t_{(l)} \le t_{(i)}} \alpha_l\right)^{\exp\{\hat{\beta} z_j\}}\right] \\
= & \prod_{1 \le i \le k} \left[\prod_{j \in D_i} \left(1 - \alpha_i^{\exp\{\hat{\beta} z_j\}}\right)\right] \left[\prod_{j \in R(t_{(i)}) - D_i} \alpha_i^{\exp\{\hat{\beta} z_j\}}\right]. \qquad (6)
\end{aligned}
$$

Differentiating the logarithm of (6) with respect to α_i gives the maximum likelihood estimate of α_i, $\hat{\alpha}_i$. If there are no ties,

$$
\hat{\alpha}_i = \left[1 - \frac{\exp\{\hat{\beta} z_{(i)}\}}{\sum_{j \in R(t_{(i)})} \exp\{\hat{\beta} z_j\}}\right]^{\exp\{-\hat{\beta} z_{(i)}\}}. \qquad (7)
$$

However, $\hat{\alpha}_i$ has no close form when ties data is presented. Finally, the maximum likelihood estimate of the baseline survival function is given by

$$\widehat{S}_{0,KP}(t) = \prod_{i|t_{(i)} \leq t} \widehat{\alpha}_i.$$

3.3 Our Proposed Estimator

Similar to Kalbfleisch & Prentice argument, we start by assuming our proposed estimator of baseline survival function has only jump points on $\{t_{(1)}, \cdots, t_{(k)}\}$. Instead of considering the full likelihood function (1), we focus on the full likelihood function (2). That is, we plug in Cox model to the hazard function and Lehmann alternative model to the survival function of each individual. Furthermore, if an observed right censoring time is tied to an observed failure time, we assume the observed right censoring time is larger than that observed failure time.

Follow the same argument as Kalbfleisch & Prentice, the full likelihood function (2) becomes

$$\begin{aligned} L(\cdot|\underset{\sim}{t}, \underset{\sim}{\delta}, \underset{\sim}{z}) &= \prod_{1 \leq i \leq k} \prod_{j \in D_i} h_0(t_{(i)}) \exp\{\hat{\beta} z_j\} \prod_{j \in D_i} S_0(t_{(i)})^{\exp\{\hat{\beta} z_j\}} \prod_{j \in C_i} S_0(t_{(i)} + \varepsilon)^{\exp\{\hat{\beta} z_j\}} \\ &= \prod_{1 \leq i \leq k} \prod_{j \in D_i} (1 - \alpha_i) \exp\{\hat{\beta} z_j\} \prod_{j \in D_i} \left(\prod_{t_{(l)} < t_{(i)}} \alpha_l \right)^{\exp\{\hat{\beta} z_j\}} \prod_{j \in C_i} \left(\prod_{t_{(l)} \leq t_{(i)}} \alpha_l \right)^{\exp\{\hat{\beta} z_j\}} \\ &= \prod_{1 \leq i \leq k} \prod_{j \in D_i} (1 - \alpha_i) \exp\{\hat{\beta} z_j\} \prod_{j \in R(t_{(i)}) - D_i} \alpha_i^{\exp\{\hat{\beta} z_j\}}. \end{aligned} \tag{8}$$

Differentiating the logarithm of (8) with respect to α_i gives the maximum likelihood estimate of α_i, $\hat{\alpha}_i$.

$$\hat{\alpha}_i = \frac{\sum_{j \in R(t_{(i)}) - D_i} \exp\{\hat{\beta} z_j\}}{1 + \sum_{j \in R(t_{(i)}) - D_i} \exp\{\hat{\beta} z_j\}}.$$

Finally, we obtain an estimator for the baseline survival which is

$$\hat{S}_{0,WW}(t) = \prod_{i|t_{(i)} < t} \left[1 - \frac{1}{1 + \sum_{j \in R(t_{(i)}) - D_i} \exp\{\hat{\beta} z_j\}} \right]. \tag{9}$$

As we can see, this estimator has a similar close form as Breslow's estimator but never gives a negative value. Thus, both of them should have the same convergence rate. Furthermore, if $\widehat{\beta} = 0$, three of the derived estimators reduce to Kaplan-Meier estimator which is the nonparametric estimator for the survival function in homogeneous group.

3.4 The Modification when Ties are Presented

If the failure times T are continuous, the probability to have data with ties among the failure times is zero. However, due to the measurement error in data recording, we may impossible to figure out the order of failure times of the patients who have the same recorded failure times but in fact happened at different time points.

Let $t_{(1)} < t_{(2)} < \cdots < t_{(k)}$ denote the k distinct, ordered, event times, d_i be the number of deaths at $t_{(i)}$, D_i be the set of all individuals who died at $t_{(i)}$ and $s_i = \sum\limits_{j \in D_i} z_j$.

There are several suggestions for constructing the partial likelihood when ties are presented in the recorded data. Two of them are given in the following.

(1) Breslow (1974) approximated the partial likelihood as

$$L_1(\beta) = \prod_{i=1}^{k} \frac{\exp(\beta s_i)}{[\sum_{j \in R(t_{(i)})} \exp(\beta z_j)]^{d_i}}$$

When there are few ties, this approximation works well, and this likelihood is implemented in most statistical packages.

(2) Efron (1977) suggested a partial likelihood of

$$L_2(\beta) = \prod_{i=1}^{k} \frac{\exp(\beta s_i)}{\prod_{j=1}^{d_i}[\sum_{k \in R(t_{(i)})} \exp(\beta z_k) - \frac{j-1}{d_i} \sum_{k \in D_i} \exp(\beta z_k)]}.$$

When the number of ties is small, Efron's and Breslow's suggested partial likelihood are close to each other.

Again, after finishing the estimate of beta, we are interested in discussing the revision of survival estimator. If there are ties, both Breslow's and our proposed estimators of baseline survival function still have close from. Specifically,

$$\hat{S}_{0,B}(t) = \prod_{i|t_{(i)}<t} \left[1 - \frac{d_i}{\sum\limits_{j \in R(t_{(i)})} \exp\{\hat{\beta} z_j\}} \right],$$

and

$$\hat{S}_{0,WW}(t) = \prod_{i|t_{(i)}<t} \left[1 - \frac{d_i}{d_i + \sum\limits_{j \in R(t_{(i)})-D_i} \exp\{\hat{\beta} z_j\}} \right].$$

In the simulation studies, we also consider Efron's ties data correction to both $\hat{S}_{0,B}(\cdot)$ and $\hat{S}_{0,WW}(\cdot)$. Then, the jump of $\hat{S}_{0,B}(\cdot)$ and $\hat{S}_{0,WW}(\cdot)$ at failure time $t_{(i)}$, are modified as

$$\prod_{1 \le j \le d_i} \left[1 - \frac{1}{\sum_{k \in R(t_{(i)})} \exp\{\hat{\beta} z_k\} - \frac{j-1}{d_i} \sum_{k \in D_i} \exp\{\hat{\beta} z_k\}} \right],$$

and

$$\prod_{1 \le j \le d_i} \left[1 - \frac{1}{1 + \sum_{k \in R(t_{(i)})-D_i} \exp\{\hat{\beta} z_k\} + \frac{d_i - j}{d_i} \sum_{k \in D_i} \exp\{\hat{\beta} z_k\}} \right].$$

4 Simulation Studies

In this section, a series of simulations were run to see the finite sample performance of Breslow's, Kalbfleisch & Prentice's and our proposed baseline survival function estimators under some specified models. We first considered a simple model M1 in which the failure times T, conditional on z, were generated from the exponential distribution with hazard function $\lambda(t) = \exp\{\beta z\}$ where $\beta = -2, -1, 0, 1, 2$, respectively, and z being an uniform random variable on $(0, 1)$. The influence of censoring proportion in survival analysis is important. In this case, the censoring times C were generated from exponential distribution with hazard rate equals to 1 and λ^*, respectively, in which λ^* is chosen so that $P(T < C) = 0.5$.

In the second model M2, we replaced the exponential distribution by Weibull distribution to the baseline survival function so that we could understand the effects of changing the baseline survival function. Here the hazard function was defined as $\lambda(t) = 0.5t^{-0.5}$.

Ties may occur in survival data because of the round-off error. We rounded off the simulated data from M1 so that number of distinct failure times was reduced about 50%. The result is referred to M3.

Finally, we considered a discrete model M4 based on the Lehmann alternative model. Specifically, the failure times T conditional on z and censoring times C were generated from the geometric distribution with the parameter $p = 1 - S_0^{\exp(\beta z)}$ and $p = 0.1$, respectively, where $S_0 = 0.9$, $\beta = -2, -1, 0, 1, 2$ and z being an uniform random variable on $(0, 1)$.

The simulations for comparing different survival estimators were done for sample size of $n = 100, 200$ and 400 with 10000 repetitions. Note that, in each table, E1, E2, KM and E3 represent Breslow's, Kalbfleisch & Prentice, Kaplan-Meier and our estimators, respectively.

Five measurements were considered to compare the performance of these four estimators. They are specified in the following. Furthermore, for each measurement, we recorded their average of 10000 runs. It is important to note that some of the median estimates did not exist since the survival function estimates were above 0.5. Therefore, we recorded only the truncated average for the median estimates.

1. The estimate of median m, $\hat{m}$. Note that m is the median if $S_0(m+) \leq 0.5$ and $S_0(m-) \geq 0.5$.

2. The estimate of truncated mean, $\hat{\theta} = \int_0^3 \hat{S}_0(u)du$.

3. The truncated mean disparity, $\hat{\delta} = \int_0^3 \hat{S}_0(u) - S_0(u)du$.

4. Accumulating the total area between the estimator and the true survival function, $\hat{\varepsilon} = \int_0^3 |\hat{S}_0(u) - S_0(u)|du$.

5. The estimate survival probability $\hat{S}_0(0.3)$ when $t = 0.3$.

We recorded both the simulation mean and simulation variance of these five measurements.

Tables 1-3 displayed the results when sample size equals 100, 200 and 400 for M1, respectively. The censoring times were generated from exp(1) for $\beta = -2, -1, 0, 1, 2$. In addition, average number of failure in each case of sample sizes 100, 200 and 400 were (28.359, 38.045, 50.055, 61.982, 71.718), (56.646, 76.025, 100.07, 124.07, 143.32) and (113.325, 151.926, 200.038, 247.926, 286.523), respectively.

Tables 4-6 also displayed the simulations of sample size equals 100, 200 and 400 for M1. However, the censoring times were generated from the exponential distribution hazard rate λ^* so that about 50% of the observation were right censoring.

According to these 6 tables, we found that the Kaplan-Meier estimator is more efficient than the others when $\beta = 0$. Furthermore, tables 1-6 showed that the estimates of median based on these three estimators were slightly over-estimated in all cases except the case with sample size 100 and $\beta = 2$ in table 4. In fact, it was truncated mean. In this case, only 81% of the 10000 simulation runs gave median estimate. Due to the format of Breslow's and our proposed estimators, it was trivial that $\hat{S}_{0,B}(t)$ will be greater than $\hat{S}_{0,WW}(t)$ when the true β was greater than zero. Therefore, the estimate of median based on our proposed estimator will be closer to the true median in this case.

Tables 7-9 displayed the simulations of sample size equals 100, 200 and 400 of M2 under different $\beta = -2, -1, 0, 1, 2$. In addition, average number of failure in each case of sample sizes 100, 200 and 400 were (39.66, 46.87, 54.50, 62.52, 69.88), (79.57, 93.76, 109.21, 125.03, 139.65), (159.33, 187.22, 218.27, 249.86, 279.10), respectively.

Basically, the conclusion between these three estimators were not much different in both M1 and M2. That is, the conclusions did not affect by the distribution of the failure times. However, the estimators of the baseline survival function with exponential distribution were closer to the true baseline survival function than that of weibull distribution. This could be known through the measurements $\overline{\delta}$ and $\overline{\varepsilon}$.

Tables 10-12 displayed the simulations of sample size equals 100, 200 and 400 of M3. When the censoring times are generated from exp(1) for $\beta = -2, -1, 0, 1, 2$. After rounding off data, average number of the reduction of failure point in each case of sample sizes 100, 200 and 400 were (25.66, 32.90, 40.02, 43.51, 41.74), (46.66, 57.68, 66.28, 66.74, 59.94), (79.05, 92.43, 98.82, 92.55, 80.02), respectively.

Tables 13-15 displayed the simulations of sample size equals 100, 200 and 400 of M3. When the censoring times are generated from $\exp(\lambda^*)$ for $\beta = -2, -1, 0, 1, 2$, in which λ^* chosen so that $P(T < c) = 0.5$. After rounding off data, average number of the reduction of failure point in each case of sample sizes 100, 200 and 400 were (45.24, 43.18, 40.00, 35.06, 28.52), (82.74, 75.89, 66.24, 53.73, 40.34), (140.72, 121.94, 98.82, 74.25, 52.73), respectively.

Tables 16-18 displayed the simulations of sample size equals 100, 200 and 400 for M4, respectively. From the results, we could see that our proposed estimator was underestimated the true median in all case. However, the simulation variance was smaller than that of Breslow's and Kalbfleisch & Prentice's estimators. It is noteworthy that the median estimates of these three estimators almost fell in $\{6, 7, 8\}$ where the true median was 7. This can be found in Tables 16.1 to 18.1. On the other hand, our proposed estimator was closer to the true baseline survival function than the other two estimators in the sense of δ and ε which described in page 13. Due to our simulation results, we could see that Kalbfleisch & Prentice's estimator was not significantly better that Breslow's estimator even in the discrete cases with Lehmann alternative model. However, Breslow's estimator has close form in these cases. Therefore, we recommended Breslow's estimator than Kalbfleisch & Prentice's estimator in the discrete cases.

Tables 22-27 were under the same conditions as Tables 10-15. The different was the baseline survival function estimate that was not revised. In this tables, E1* represented Breslow's estimator and E3* represented our proposed estimator without modifying. The result of the simulation studies show that if tie happened due to the rounding error, the modified estimator will give a smaller bias than that without modifying.

5 Conclusion

In the homogeneous case, that is, where $\beta = 0$, Breslow's, Kalbfleisch & Prentice's and our proposed estimators of the baseline survival function will reduce to Kaplan-Meier (1958) estimator. However, through the results of simulation studies, we found that Kaplan-Meier estimator is much better than these three estimators when the data is generated from a homogeneous population. Therefore, Kaplan-Meier estimator is still be recommended in this case.

Furthermore, Breslow's and our proposed estimators have a close form in all cases, but Kalbfleisch & Prentice's estimators requires an iterative method for estimating when some of the failure times are ties. Moreover, even discrete cases with Lehmann alternative model, Kalbfleisch & Prentice's estimator does not show significantly better than Breslow's and ours estimators in the limited simulation studies. Thus, Kalbfleisch & Prentice's estimator is not suggest recommended.

Clearly, our proposed estimator has a similar close form as Berslow's estimator but never gives a negative value. To compare the biasness of Breslow's and ours proposed estimators, the sign of true β will provide opposite conclusion in our simulation studies. However, in all the results of simulation studies with continuous survival times, our proposed estimator tends to give a smaller simulation variance. Although our proposed estimator does not work well in the discrete cases, the performance will become better and better when sample size increase.

Typically, ties data occurs because of the round-off errors. In this case, the modified Breslow's and our proposed estimators which based on Efron's adjustment of ties gave smaller bias than that without modification.

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Table 1: Summary statistics on M1 when sample size equals 100.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.36097(6.1647)	9.69828(5.8538)	1.92757(2.1764)	2.25262(1.6189)	7.37150(5.640)
	E2	7.34794(6.0076)	9.81014(5.6311)	1.89046(2.1518)	2.21362(1.6404)	7.37226(5.590)
	E3	7.34051(5.8845)	9.64682(5.1897)	1.80922(1.9370)	2.19428(1.4567)	7.37300(5.542)
	KM	7.06275(1.6744)	9.59646(1.6590)	1.01973(0.6279)	1.51499(0.4114)	7.40440(2.234)
1	E1	7.35338(5.4296)	9.98540(5.4818)	1.87547(2.1977)	2.25617(1.6074)	7.39173(4.828)
	E2	7.27895(5.0276)	10.00832(5.3078)	1.85847(2.1098)	2.33011(1.5019)	7.38385(4.810)
	E3	7.21360(4.7335)	9.39948(4.8164)	1.75928(1.7316)	2.30392(1.1882)	7.37596(4.792)
2	E2	7.35150(5.4029)	10.36165(5.6835)	1.98383(2.4864)	2.44328(1.8221)	7.40006(4.774)
	E2	7.18242(4.6243)	10.30967(6.0876)	2.06360(2.4810)	2.68301(1.6395)	7.38004(4.784)
	E3	7.04319(4.2788)	9.21265(5.3511)	1.89243(1.8533)	2.55717(1.1659)	7.35976(4.794)
-1	E2	7.34414(7.0763)	9.60264(6.6831)	2.05800(2.4574)	2.37587(1.8390)	7.34964(6.998)
	E2	7.38074(7.0363)	9.75232(6.5012)	2.02874(2.4476)	2.32676(1.8580)	7.35836(6.897)
	E3	7.41388(7.0580)	9.82761(6.2626)	1.99180(2.4009)	2.30235(1.8288)	7.36688(6.801)
-2	E1	7.37528(8.4944)	9.58817(7.6300)	2.19454(2.8209)	2.56229(2.0800)	7.33455(8.795)
	E2	7.44015(8.5123)	9.81589(7.4874)	2.17089(2.8726)	2.51411(2.1551)	7.35132(8.618)
	E3	7.50482(8.5926)	9.99178(7.3282)	2.15758(2.9124)	2.49670(2.2098)	7.36748(8.452)

Table 2: Summary statistics on M1 when sample size equals 200.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.12698(2.5224)	9.55914(2.7415)	1.31976(1.0028)	1.56657(7.329)	7.39924(2.683)
	E2	7.12397(2.4977)	9.60296(2.6747)	1.30051(0.9934)	1.53900(7.319)	7.39942(2.672)
	E3	7.12167(2.4725)	9.54383(2.5545)	1.27234(0.9372)	1.54205(6.893)	7.39959(2.661)
	KM	6.99525(0.7961)	9.51696(0.8212)	0.72087(0.3018)	1.07884(1.842)	7.41396(1.124)
1	E1	7.11740(2.3650)	9.69926(2.7032)	1.31116(1.0228)	1.62715(7.224)	7.39157(2.416)
	E2	7.08798(2.3060)	9.71344(2.6454)	1.29567(1.0111)	1.64355(7.129)	7.38751(2.412)
	E3	7.06208(2.2574)	9.36059(2.5405)	1.28335(0.9134)	1.68350(6.304)	7.38346(2.408)
2	E1	7.14326(2.2366)	9.94216(2.8535)	1.36905(1.1726)	1.77928(8.177)	7.40713(2.328)
	E2	7.06658(2.1066)	9.92650(2.9612)	1.40290(1.1730)	1.88004(7.875)	7.39707(2.331)
	E3	6.99560(1.9913)	9.26861(2.8515)	1.37718(1.0092)	1.88820(6.486)	7.38695(2.334)
-1	E1	7.11866(2.9765)	9.52425(3.1427)	1.40780(1.1610)	1.64219(8.540)	7.37717(3.244)
	E2	7.13713(2.9737)	9.60174(3.0849)	1.39233(1.1560)	1.61876(8.566)	7.38118(3.223)
	E3	7.15317(2.9820)	9.65835(3.0271)	1.37929(1.1488)	1.60728(8.545)	7.38514(3.202)
-2	E1	7.12644(3.5084)	9.51359(3.5961)	1.50833(1.3209)	1.77218(9.526)	7.37822(4.105)
	E2	7.16197(3.5341)	9.63627(3.5524)	1.49601(1.3321)	1.74950(9.731)	7.38597(4.067)
	E3	7.19545(3.5524)	9.74184(3.5144)	1.49129(1.3477)	1.74101(9.932)	7.39360(4.029)

Table 3: Summary statistics on M1 when sample size equals 400.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.02636(1.1860)	9.52635(1.3927)	0.94391(5.022)	1.11735(3.607)	7.40001(1.343)
	E2	7.02547(1.1801)	9.53925(1.3638)	0.93319(4.942)	1.10278(3.546)	7.40005(1.340)
	E3	7.02487(1.1752)	9.52258(1.3406)	0.92583(4.837)	1.10624(3.468)	7.40010(1.338)
	KM	6.96353(0.3885)	9.50528(0.4111)	0.51062(1.503)	0.76697(0.888)	7.40762(0.577)
1	E1	7.01308(1.1014)	9.56689(1.3609)	0.92837(5.031)	1.17653(3.411)	7.39764(1.239)
	E2	6.99946(1.0885)	9.56091(1.3304)	0.91834(4.904)	1.17004(3.348)	7.39559(1.238)
	E3	6.98657(1.0782)	9.37378(1.3138)	0.92446(4.756)	1.21508(3.183)	7.39355(1.237)
2	E1	7.02577(0.9871)	9.70997(1.4057)	0.95613(5.346)	1.28767(3.524)	7.40506(1.138)
	E2	6.99295(0.9662)	9.68109(1.4253)	0.95731(5.408)	1.32075(3.610)	7.40001(1.139)
	E3	6.95900(0.9430)	9.31121(1.4292)	0.97622(5.125)	1.36531(3.318)	7.39494(1.139)
-1	E1	7.01845(1.3643)	9.51941(1.5921)	1.00595(5.803)	1.16314(4.286)	7.39135(1.612)
	E2	7.02794(1.3638)	9.55738(1.5756)	1.00018(5.782)	1.15488(4.285)	7.39327(1.606)
	E3	7.03647(1.3642)	9.59113(1.5609)	0.99582(5.770)	1.14983(4.288)	7.39517(1.601)
-2	E1	7.02989(1.6383)	9.50392(1.8239)	1.07723(6.633)	1.25339(4.828)	7.39615(1.964)
	E2	7.04718(1.6453)	9.56599(1.8112)	1.07210(6.658)	1.24518(4.873)	7.39989(1.955)
	E3	7.06432(1.6543)	9.62317(1.8005)	1.06996(6.703)	1.24145(4.929)	7.40361(1.946)

Table 4: Summary statistics on M1 when sample size equals 100 and $P(T > C) = 0.5$.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.31284(5.7233)	9.67677(0.56682)	1.89462(2.1088)	2.22300(1.5638)	7.38556(5.381)
	E2	7.30328(5.5737)	9.79377(0.54308)	1.85553(2.0726)	2.18075(1.5773)	7.38630(5.336)
	E3	7.29048(5.3841)	9.62613(0.50164)	1.77791(1.8705)	2.16728(1.4037)	7.38701(5.291)
	KM	7.06472(1.6595)	9.58873(0.17055)	1.03589(0.6398)	1.52870(0.4121)	7.41773(2.281)
1	E1	7.34021(7.0448)	10.67494(0.95511)	2.57704(4.2849)	3.02761(3.2963)	7.36653(5.841)
	E2	7.20378(6.0425)	10.76971(0.96297)	2.63281(4.3042)	3.22964(3.0743)	7.35796(5.815)
	E3	7.16759(6.1282)	9.81720(0.90148)	2.40144(3.3466)	3.06360(2.2910)	7.34939(5.788)
2	E1	6.90155(5.7722)	13.12329(1.60379)	4.39610(9.8244)	4.87227(8.2190)	7.38127(6.605)
	E2	6.54651(4.3469)	12.88535(1.92155)	4.67809(8.7762)	5.33199(6.3185)	7.35381(6.625)
	E3	6.59133(4.8325)	10.99241(2.07386)	4.00343(6.9307)	4.71337(4.9342)	7.32574(6.643)
-1	E1	7.22182(4.8663)	9.53294(0.47802)	1.74726(1.7279)	1.96991(1.3413)	7.36598(5.927)
	E2	7.25051(4.8521)	9.61363(0.46725)	1.72451(1.7107)	1.94283(1.3322)	7.37388(5.855)
	E3	7.27681(4.8360)	9.68284(0.45704)	1.70566(1.6935)	1.92413(1.3225)	7.38161(5.785)
-2	E1	7.14103(4.7765)	9.49454(0.44806)	1.69365(1.6120)	1.91632(1.2394)	7.36362(6.494)
	E2	7.19456(4.8038)	9.59710(0.44179)	1.67793(1.6112)	1.89906(1.2420)	7.37795(6.394)
	E3	7.24937(4.8250)	9.69420(0.43604)	1.66741(1.6168)	1.88766(1.2492)	7.39186(6.299)

Table 5: Summary statistics on M1 when sample size equals 200 and $P(T > C) =$ 0.5.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.13145(2.5450)	9.57839(0.27856)	1.32723(1.0297)	1.57339(0.7497)	7.39100(2.675)
	E2	7.12942(2.5210)	9.62297(0.27156)	1.30915(1.0162)	1.54598(0.7480)	7.39118(2.664)
	E3	7.12706(2.4958)	9.56357(0.25942)	1.28005(0.9593)	1.54811(0.7030)	7.39135(2.653)
	KM	7.00969(0.8063)	9.53679(0.08104)	0.71997(0.2932)	1.07786(1.784)	7.40667(1.144)
1	E1	7.19676(3.3789)	10.21282(0.45824)	1.77417(1.9396)	2.19607(1.3966)	7.39186(2.849)
	E2	7.14646(3.1135)	10.31045(0.46806)	1.82736(1.9945)	2.33848(1.3757)	7.38738(2.843)
	E3	7.11176(3.0705)	9.62465(0.44882)	1.70356(1.6008)	2.28055(1.0537)	7.38290(2.837)
2	E1	7.23153(4.3895)	12.22226(1.00530)	3.38450(5.9969)	3.84133(4.9018)	7.40090(3.295)
	E2	7.00869(3.5050)	12.11218(1.16965)	3.60801(5.4905)	4.22051(3.8505)	7.38710(3.301)
	E3	6.96628(3.6103)	10.64024(1.31157)	3.16589(4.3873)	3.81356(3.0571)	7.37317(3.306)
-1	E1	7.07850(2.1932)	9.52607(0.23459)	1.22425(0.8476)	1.38017(0.6515)	7.38843(2.785)
	E2	7.09182(2.1877)	9.56631(0.23173)	1.21603(0.8425)	1.37101(0.6486)	7.39209(2.769)
	E3	7.10544(2.1858)	9.60436(0.22909)	1.20930(0.8388)	1.36381(0.6465)	7.39572(2.753)
-2	E1	7.04850(2.1877)	9.51669(0.22079)	1.18307(0.8083)	1.33987(0.6251)	7.39039(3.180)
	E2	7.07444(2.1905)	9.56821(0.21916)	1.17783(0.8085)	1.33419(0.6259)	7.39722(3.156)
	E3	7.10150(2.1959)	9.61834(0.21760)	1.17420(0.8106)	1.33038(0.6281)	7.40395(3.133)

Table 6: Summary statistics on M1 when sample size equals 400 and $P(T > C) =$ 0.5.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.00359(1.1665)	9.51220(1.3761)	0.93753(0.4971)	1.11309(0.3569)	7.39313(1.314)
	E2	7.00332(1.1626)	9.52554(1.3492)	0.92782(0.4888)	1.09872(0.3515)	7.39318(1.311)
	E3	7.00259(1.1570)	9.50890(1.3238)	0.91936(0.4786)	1.10220(0.3436)	7.39324(1.308)
	KM	6.96147(0.3879)	9.51260(0.4144)	0.51326(0.1510)	0.76758(0.0905)	7.40641(0.561)
1	E1	7.04037(1.4204)	9.90250(2.3521)	1.24347(0.9660)	1.60420(0.6650)	7.39630(1.442)
	E2	7.02214(1.3975)	9.94392(2.4182)	1.27556(0.9862)	1.68914(0.6625)	7.39402(1.441)
	E3	7.00227(1.3662)	9.51001(2.3905)	1.23886(0.8557)	1.69351(0.5568)	7.39174(1.440)
2	E1	7.19670(2.4599)	11.50217(6.2071)	2.58832(3.5076)	3.01997(2.7980)	7.40161(1.577)
	E2	7.09923(2.1732)	11.47891(7.2475)	2.81280(3.2429)	3.34527(2.2572)	7.39471(1.579)
	E3	7.03552(2.1450)	10.34762(8.2302)	2.49756(2.7067)	3.07271(1.8543)	7.38777(1.581)
-1	E1	7.01836(1.1073)	9.53269(1.2007)	0.87425(0.4372)	0.98442(0.3375)	7.40312(1.402)
	E2	7.02515(1.1076)	9.55283(1.1931)	0.87150(0.4361)	0.98136(0.3368)	7.40489(1.398)
	E3	7.03208(1.1061)	9.57245(1.1858)	0.86916(0.4353)	0.97884(0.3364)	7.40664(1.394)
-2	E1	6.97727(1.0345)	9.49616(1.0708)	0.82685(0.3871)	0.93789(0.2974)	7.39453(1.554)
	E2	6.98960(1.0378)	9.52207(1.0669)	0.82465(0.3871)	0.93557(0.2976)	7.39788(1.549)
	E3	7.00305(1.0404)	9.54763(1.0630)	0.82302(0.3876)	0.93387(0.2982)	7.40121(1.543)

Table 7: Summary statistics on M2 when sample size equals 100.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	5.67272(13.0376)	1.03917(8.1456)	2.86042(4.6329)	6.65298(5.5961)	5.74698(8.567)
	E2	5.64298(12.4475)	1.04312(7.8206)	2.84481(4.5695)	6.66222(5.5110)	5.74778(8.477)
	E3	5.64658(12.5236)	1.03785(7.5949)	2.78886(4.4297)	6.63195(5.3158)	5.74854(8.390)
	KM	5.08577(3.0457)	1.03629(2.5295)	2.25567(1.9874)	6.48578(2.0476)	5.77890(2.758)
1	E1	5.63183(11.0262)	1.05182(7.1966)	2.83171(4.4101)	6.73932(5.0598)	5.77847(7.462)
	E2	5.53062(9.9943)	1.04389(6.9496)	2.75582(4.2307)	6.70925(4.8020)	5.76483(7.412)
	E3	5.47282(9.6911)	1.00825(6.9493)	2.55338(3.8584)	6.48554(4.4762)	5.75119(7.362)
2	E1	5.61115(11.1584)	1.07246(7.1693)	2.97450(4.5406)	6.95921(5.1052)	5.76887(7.316)
	E2	5.39306(9.4968)	1.05274(7.2868)	2.88294(4.2497)	6.91872(4.5973)	5.73181(7.318)
	E3	5.23684(8.9676)	0.97920(7.431)	2.55059(3.5747)	6.45977(4.1461)	5.69415(7.318)
-1	E1	5.70268(14.7385)	1.03414(9.3902)	2.93908(5.2066)	6.64749(6.3183)	5.73534(10.133)
	E2	5.74236(14.8047)	1.04616(9.0667)	2.96804(5.2339)	6.71572(6.2983)	5.74774(9.982)
	E3	5.78393(14.8484)	1.05481(8.7944)	2.98699(5.2422)	6.77778(6.2241)	5.75984(9.838)
-2	E1	5.84587(17.8392)	1.03907(11.2072)	3.13103(6.0687)	6.77281(7.3295)	5.73890(12.225)
	E2	5.91766(18.0306)	1.05895(10.7940)	3.18577(6.2080)	6.89081(7.3863)	5.76264(11.977)
	E3	5.99320(18.2909)	1.07514(10.4803)	3.24202(6.3228)	7.00575(7.3802)	5.78557(11.745)

Table 8: Summary statistics on M2 when sample size equals 200.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	5.28425(5.5707)	1.03564(4.1168)	2.40224(2.8642)	6.45487(3.1808)	5.77366(4.233)
	E2	5.27914(5.5087)	1.03628(4.0301)	2.39616(2.8340)	6.45502(3.1352)	5.77383(4.212)
	E3	5.27487(5.4588)	1.03542(3.9598)	2.38152(2.7971)	6.45039(3.0748)	5.77401(4.190)
	KM	4.93688(1.3300)	1.03268(1.2438)	2.11693(1.1556)	6.38227(1.0829)	5.78369(1.375)
1	E1	5.22912(4.7829)	1.03896(3.7364)	2.39167(2.6768)	6.49984(2.8519)	5.77336(3.778)
	E2	5.19108(4.6425)	1.03268(3.6519)	2.33510(2.5925)	6.45701(2.7635)	5.76649(3.766)
	E3	5.15330(4.4711)	1.01518(3.6962)	2.21780(2.4680)	6.33998(2.6734)	5.75962(3.754)
2	E1	5.24694(4.5081)	1.05118(3.6920)	2.49427(2.6738)	6.63650(2.8161)	5.78939(3.473)
	E2	5.15514(4.1869)	1.03705(3.6892)	2.39256(2.5432)	6.56311(2.6390)	5.77094(3.474)
	E3	5.06925(3.8941)	0.99771(3.9893)	2.16613(2.3466)	6.30285(2.5557)	5.75235(3.474)
-1	E1	5.29010(6.5032)	1.03183(4.6407)	2.42224(3.1310)	6.42938(3.5598)	5.75751(4.918)
	E2	5.31042(6.5237)	1.03770(4.5495)	2.45188(3.1442)	6.47792(3.5221)	5.76337(4.883)
	E3	5.33028(6.5364)	1.04281(4.4732)	2.47906(3.1557)	6.52324(3.4817)	5.76917(4.848)
-2	E1	5.38640(8.2990)	1.03119(5.4714)	2.51589(3.4728)	6.45417(4.0857)	5.75052(5.954)
	E2	5.42403(8.4022)	1.04160(5.3508)	2.56884(3.5271)	6.53887(4.0648)	5.76189(5.896)
	E3	5.46381(8.4898)	1.05082(5.2552)	2.62076(3.5734)	6.61907(4.0331)	5.77307(5.839)

Table 9: Summary statistics on M2 when sample size equals 400.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	5.03040(2.2551)	1.03590(2.0386)	2.20068(1.7250)	6.41115(1.6411)	5.77846(2.122)
	E2	5.03016(2.2490)	1.03598(2.0169)	2.19927(1.7128)	6.41109(1.6245)	5.77850(2.117)
	E3	5.02860(2.2350)	1.03585(1.9983)	2.19646(1.7013)	6.41043(1.6085)	5.77853(2.111)
	KM	4.86896(0.6279)	1.03361(0.6207)	2.10642(0.6162)	6.37368(0.5525)	5.78058(0.706)
1	E1	5.01839(1.9039)	1.03735(1.8053)	2.19645(1.5722)	6.42771(1.4555)	5.78796(1.831)
	E2	5.00377(1.8785)	1.03269(1.7879)	2.15318(1.5456)	6.38659(1.4375)	5.78449(1.828)
	E3	4.98862(1.8546)	1.02519(1.7993)	2.08886(1.5210)	6.32762(1.4220)	5.78102(1.825)
2	E1	4.99196(1.7774)	1.03945(1.8884)	2.22324(1.6274)	6.46838(1.5177)	5.78291(1.746)
	E2	4.95343(1.7278)	1.02945(1.8925)	2.13780(1.5816)	6.39679(1.4672)	5.77372(1.746)
	E3	4.91383(1.6687)	1.01061(2.0058)	1.99434(1.5452)	6.25873(1.4629)	5.76449(1.746)
-1	E1	5.05883(2.7341)	1.03428(2.3537)	2.21322(1.9159)	6.40093(1.8923)	5.77339(2.488)
	E2	5.06870(2.7393)	1.03712(2.3305)	2.23504(1.9164)	6.42774(1.8756)	5.77625(2.479)
	E3	5.07909(2.7435)	1.03981(2.3095)	2.25613(1.9167)	6.45334(1.8598)	5.77908(2.470)
-2	E1	5.11533(3.5192)	1.03334(2.8504)	2.25420(2.1900)	6.39842(2.2771)	5.76911(3.083)
	E2	5.13491(3.5467)	1.03841(2.8192)	2.29175(2.2038)	6.44593(2.2579)	5.77464(3.068)
	E3	5.15356(3.5611)	1.04319(2.7917)	2.32812(2.2163)	6.49105(2.2395)	5.78013(3.054)

Table 10: Summary statistics on M3 when sample size equals 100.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.39475(6.1460)	10.00862(6.1275)	1.99638(2.3982)	2.23481(1.9032)	7.34622(5.670)
	E2	7.38143(5.9842)	10.00004(5.7670)	1.93435(2.2728)	2.17922(1.7903)	7.34698(5.620)
	E3	7.29747(5.7564)	9.90694(5.4257)	1.86632(2.1061)	2.11862(1.6398)	7.32555(5.645)
	KM	7.10089(1.7001)	992918(1.5199)	1.01559(0.6707)	1.41773(0.4697)	7.37891(2.247)
1	E1	7.34265(5.2372)	10.72609(5.3956)	2.05357(2.6762)	2.36396(2.1012)	7.35603(4.854)
	E2	7.27243(4.8887)	10.51239(5.0581)	1.92177(2.3852)	2.22751(1.8782)	7.34807(4.836)
	E3	7.13646(4.5321)	10.22763(4.6947)	1.77798(2.0596)	2.08893(1.6080)	7.31754(4.883)
2	E1	7.23062(4.7441)	11.47476(5.0570)	2.38170(3.2757)	2.80105(2.5085)	7.33573(4.819)
	E2	7.09483(4.3336)	11.07405(4.8192)	2.11822(2.8031)	2.52743(2.2079)	7.31528(4.828)
	E3	6.89614(3.9394)	10.59798(4.5253)	1.86121(2.2618)	2.26605(1.8326)	7.27192(4.902)
-1	E1	7.39186(7.1117)	9.70179(6.8136)	2.08049(2.5246)	2.37150(1.9087)	7.32771(7.005)
	E2	7.42430(7.0768)	9.83106(6.5703)	2.04417(2.4994)	2.32389(1.9118)	7.33644(6.905)
	E3	7.38225(6.9148)	9.85736(6.3181)	2.00576(2.4208)	2.28370(1.8528)	7.32267(6.897)
-2	E1	7.42037(8.4867)	9.65068(7.6889)	2.20073(2.8673)	2.56393(2.1191)	7.31454(8.805)
	E2	7.48343(8.5514)	9.87295(7.5265)	2.17849(2.9177)	2.51873(2.1955)	7.33131(8.630)
	E3	7.47546(8.4846)	9.98036(7.3238)	2.15625(2.9027)	2.48811(2.2026)	7.32568(8.573)

Table 11: Summary statistics on M3 when sample size equals 200.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.16227(2.5294)	9.70615(2.8009)	1.33446(1.0616)	1.53373(0.7939)	7.37375(2.695)
	E2	7.15867(2.5022)	9.70304(2.6999)	1.30959(1.0251)	1.51327(0.7612)	7.37393(2.684)
	E3	7.08451(2.4286)	9.61862(2.5891)	1.27982(0.9646)	1.48780(0.7081)	7.35187(2.709)
	KM	7.03023(0.8082)	9.66725(0.7703)	0.70577(0.2994)	1.02385(0.1862)	7.38821(1.134)
1	E1	7.11900(2.3443)	10.10926(2.5832)	1.34387(1.1457)	1.59810(0.8614)	7.35424(2.431)
	E2	7.09189(2.2909)	9.97662(2.4841)	1.28806(1.0500)	1.53905(0.7877)	7.35015(2.427)
	E3	6.99506(2.1977)	9.77174(2.3674)	1.22689(0.9347)	1.48231(0.6892)	7.32357(2.457)
2	E1	7.06624(2.1528)	10.57914(2.4594)	1.49197(1.3933)	1.85682(1.0424)	7.34180(2.359)
	E2	6.99435(2.0191)	10.32649(2.3810)	1.36298(1.2027)	1.71643(0.9229)	7.33151(2.362)
	E3	6.85886(1.8962)	10.00019(2.2717)	1.23640(0.9909)	1.59078(0.7663)	7.29863(2.397)
-1	E1	7.16635(3.0191)	9.58367(3.1623)	1.40976(1.1814)	1.63975(0.8693)	7.35335(3.254)
	E2	7.18220(3.0138)	9.65702(3.0973)	1.39473(1.1758)	1.61865(0.8719)	7.35737(3.233)
	E3	7.12368(2.9228)	9.64073(3.0167)	1.37609(1.1421)	1.59792(0.8461)	7.33917(3.255)
-2	E1	7.17311(3.5446)	9.56693(3.6055)	1.50847(1.3340)	1.77300(0.9645)	7.35537(4.115)
	E2	7.20758(3.5744)	9.68913(3.5604)	1.49894(1.3483)	1.75254(0.9883)	7.36314(4.077)
	E3	7.16894(3.5127)	9.71661(3.4906)	1.48508(1.3309)	1.73504(0.9782)	7.34870(4.093)

Table 12: Summary statistics on M3 when sample size equals 400.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.06071(1.1915)	9.59691(1.3871)	0.94198(5.086)	1.09873(3.678)	7.37397(1.348)
	E2	7.06022(1.1863)	9.59589(1.3591)	0.93232(4.986)	1.09163(3.591)	7.37402(1.345)
	E3	6.98913(1.1548)	9.51479(1.3221)	0.91883(4.780)	1.08055(3.409)	7.35180(1.361)
	KM	6.99788(0.3917)	9.57803(0.3974)	0.50331(1.498)	0.74684(0.878)	7.38157(0.578)
1	E1	7.01494(1.0884)	9.76782(1.2946)	0.92146(5.160)	1.12123(3.650)	7.36175(1.245)
	E2	7.00210(1.0783)	9.68844(1.2631)	0.90037(4.871)	1.10120(3.405)	7.35969(1.244)
	E3	6.92194(1.0484)	9.53340(1.2173)	0.87788(4.475)	1.08287(3.063)	7.33516(1.260)
2	E1	6.95377(0.9566)	10.04087(1.2203)	0.96488(5.795)	1.25504(4.170)	7.33896(1.152)
	E2	6.92062(0.9349)	9.88563(1.1985)	0.91027(5.169)	1.19383(3.746)	7.33376(1.153)
	E3	6.82182(0.8975)	9.65617(1.1600)	0.86037(4.434)	1.14499(3.188)	7.30597(1.170)
-1	E1	7.06441(1.3735)	9.57132(1.5950)	1.00644(5.867)	1.16379(4.343)	7.36824(1.615)
	E2	7.07268(1.3741)	9.60894(1.5780)	1.00178(5.857)	1.15658(4.353)	7.37016(1.610)
	E3	7.01055(1.3514)	9.56356(1.5481)	0.99136(5.690)	1.14533(4.213)	7.34977(1.627)
-2	E1	7.07664(1.6619)	9.55711(1.8289)	1.07749(6.709)	1.25452(4.891)	7.37303(1.974)
	E2	7.09262(1.6628)	9.61910(1.8160)	1.07440(6.753)	1.24783(4.956)	7.37679(1.965)
	E3	7.03994(1.6326)	9.59720(1.7889)	1.06571(6.621)	1.23771(4.857)	7.35831(1.984)

Table 13: Summary statistics on M3 when sample size equals 100 and $P(T > C) = 0.5$.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.35031(5.7534)	10.00045(5.9254)	1.95343(2.3575)	2.19880(1.8623)	7.36017(5.413)
	E2	7.33523(5.5366)	9.99263(5.5759)	1.89290(2.2330)	2.14464(1.7504)	7.36090(5.367)
	E3	7.25041(5.2770)	9.89947(5.2490)	1.82684(2.0692)	2.08615(1.6017)	7.33934(5.395)
	KM	7.09857(1.6801)	9.93627(1.5490)	1.02671(0.6832)	1.42414(0.4767)	7.39234(2.295)
1	E1	7.23428(5.6414)	11.85974(8.7318)	2.98658(5.3701)	3.39370(4.2929)	7.33739(5.851)
	E2	7.18020(5.3811)	11.59407(8.2961)	2.78389(4.9220)	3.18614(3.9918)	7.32877(5.824)
	E3	6.99033(4.9243)	11.19274(7.7780)	2.52142(4.2783)	2.93411(3.5018)	7.28274(5.925)
2	E1	6.36459(3.3713)	14.99466(10.7720)	5.58210(9.7799)	5.99950(7.9388)	7.33608(6.626)
	E2	6.31105(3.2246)	14.43875(10.8881)	5.06670(9.5867)	5.50345(7.9390)	7.30832(6.646)
	E3	6.13565(3.0593)	13.59506(10.9060)	4.32584(8.9450)	4.81502(7.5958)	7.21707(6.907)
-1	E1	7.25171(4.8707)	9.57067(4.7844)	1.74649(1.7385)	1.96735(1.3484)	7.34059(5.970)
	E2	7.27857(4.8520)	9.64985(4.6721)	1.72416(1.7209)	1.94155(1.3400)	7.34851(5.897)
	E3	7.26474(4.8010)	9.67348(4.5487)	1.70124(1.6835)	1.91730(1.3110)	7.34284(5.874)
-2	E1	7.16497(4.7962)	9.52184(4.4810)	1.69228(1.6174)	1.91540(1.2439)	7.33415(6.504)
	E2	7.21947(4.8257)	9.62418(4.4181)	1.67776(1.6178)	1.89912(1.2480)	7.34857(6.404)
	E3	7.24664(4.8051)	9.69247(4.3470)	1.66486(1.6112)	1.88537(1.2444)	7.35454(6.340)

Table 14: Summary statistics on M3 when sample size equals 200 and $P(T > C) = 0.5$.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.16592(2.5486)	9.72321(2.8313)	1.34421(1.0731)	1.54178(0.8059)	7.36458(2.683)
	E2	7.16447(2.5271)	9.72027(2.7298)	1.31944(1.0363)	1.52148(0.7728)	7.36475(2.672)
	E3	7.09124(2.4653)	9.63685(2.6201)	1.28846(0.9780)	1.49543(0.7199)	7.34262(2.697)
	KM	7.04424(0.8141)	9.68728(0.7634)	0.70696(0.2978)	1.02643(0.1838)	7.38003(1.145)
1	E1	7.20485(3.1314)	11.06821(4.2370)	2.04457(2.5092)	2.40613(1.9356)	7.36258(2.855)
	E2	7.16276(3.0111)	10.88109(4.1022)	1.92191(2.3098)	2.27874(1.8104)	7.35807(2.849)
	E3	7.00961(2.8133)	10.55928(3.9016)	1.74210(1.9840)	2.11339(1.5662)	7.31656(2.907)
2	E1	6.99366(3.0461)	13.72074(6.6237)	4.28341(6.0727)	4.68275(4.8590)	7.35527(3.299)
	E2	6.90981(2.9365)	13.28992(6.8166)	3.89374(6.0026)	4.30822(4.9323)	7.34128(3.304)
	E3	6.66174(2.6428)	12.59454(6.9365)	3.30505(5.5760)	3.77904(4.7112)	7.26540(3.434)
-1	E1	7.10756(2.2003)	9.56093(2.3432)	1.22297(0.8509)	1.37944(0.6544)	7.36154(2.799)
	E2	7.12016(2.1997)	9.60107(2.3146)	1.21555(0.8466)	1.37097(0.6524)	7.36522(2.783)
	E3	7.09300(2.1751)	9.59124(2.2755)	1.20483(0.8317)	1.35996(0.6400)	7.35537(2.789)
-2	E1	7.07341(2.1946)	9.54438(2.2076)	1.18241(0.8112)	1.33956(0.6276)	7.36061(3.204)
	E2	7.10007(2.2045)	9.59577(2.1912)	1.17793(0.8123)	1.33460(0.6292)	7.36748(3.180)
	E3	7.10052(2.1934)	9.61697(2.1687)	1.17218(0.8077)	1.32871(0.6255)	7.36615(3.171)

Table 15: Summary statistics on M3 when sample size equals 400 and $P(T > C) = 0.5$.

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.03877(1.1731)	9.58216(1.3763)	0.93777(0.5032)	1.09577(0.3637)	7.36756(1.324)
	E2	7.03862(1.1689)	9.58142(1.3485)	0.92811(0.4933)	1.08862(0.3553)	7.36761(1.321)
	E3	6.96828(1.1397)	9.50085(1.3111)	0.91534(0.4732)	1.07787(0.3378)	7.34530(1.336)
	KM	6.99660(0.3922)	9.58384(0.4014)	0.50701(0.1510)	0.74885(0.0901)	7.38081(0.565)
1	E1	7.06651(1.4135)	10.46374(2.1376)	1.37395(1.1744)	1.68283(0.8782)	7.36748(1.445)
	E2	7.04945(1.3954)	10.33408(2.0968)	1.30435(1.0875)	1.60938(0.8240)	7.36519(1.443)
	E3	6.91612(1.3213)	10.07151(2.0159)	1.19000(0.9239)	1.51170(0.6981)	7.32598(1.474)
2	E1	7.13938(2.1127)	12.68426(4.0222)	3.23882(3.6581)	3.61161(2.9266)	7.35549(1.579)
	E2	7.06618(1.9911)	12.36726(4.1763)	2.96014(3.6227)	3.34589(2.9806)	7.34848(1.580)
	E3	6.80425(1.7657)	11.79239(4.2351)	2.49133(3.2735)	2.94638(2.7757)	7.27959(1.642)
-1	E1	7.04818(1.1105)	9.56809(1.1991)	0.87383(0.4398)	0.98446(0.3397)	7.37588(1.411)
	E2	7.05498(1.1114)	9.58817(1.1915)	0.87156(0.4392)	0.98188(0.3395)	7.37766(1.407)
	E3	7.02115(1.0991)	9.55956(1.1781)	0.86601(0.4313)	0.97608(0.3328)	7.36594(1.415)
-2	E1	7.00074(1.0404)	9.52427(1.0702)	0.82589(0.3885)	0.93738(0.2985)	7.36555(1.571)
	E2	7.01310(1.0434)	9.55011(1.0662)	0.82428(0.3890)	0.93557(0.2992)	7.36892(1.565)
	E3	6.99979(1.0365)	9.54637(1.0591)	0.82143(0.3862)	0.93267(0.2968)	7.36409(1.568)

Table 16: Summary statistics on M4 when sample size equals 100.

β	method	$\overline{m} \times 10^0$ (σ^2 of $m \times 10^0$)	$\overline{\theta} \times 10^0$ (σ^2 of $\theta \times 10^0$)	$\overline{\delta} \times 10^0$ (σ^2 of $\delta \times 10^0$)	$\overline{\varepsilon} \times 10^0$ (σ^2 of $\varepsilon \times 10^0$)	$S(t_i = 7)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.42726(4.54855)	10.12824(5.16792)	1.84294(2.07607)	2.14837(1.51797)	4.77908(10.364)
	E2	7.41897(4.45837)	10.12044(4.88571)	1.790494(1.97587)	2.10474(1.43096)	4.77973(10.217)
	E3	6.76238(3.62807)	9.31976(4.18600)	1.657784(1.50319)	1.98084(1.03161)	4.43297(10.472)
	KM	7.20950(1.49976)	10.06515(1.42997)	1.00552(0.65802)	1.47890(0.41391)	4.79116(3.623)
1	E1	7.08127(3.65509)	10.49557(4.41240)	1.78480(2.07208)	2.32511(1.33419)	4.62246(9.016)
	E2	7.02691(3.44785)	10.31133(4.17109)	1.68725(1.86461)	2.22990(1.19999)	4.59910(8.923)
	E3	6.38720(2.75115)	9.33693(3.50143)	1.50266(1.30042)	2.06355(0.78482)	4.23452(9.147)
2	E1	6.43459(2.87398)	10.60152(3.87215)	1.71356(1.98717)	2.67513(1.11272)	4.28357(8.578)
	E2	6.32970(2.68615)	10.27466(3.71073)	1.58243(1.69444)	2.54307(0.98486)	4.21674(8.583)
	E3	5.73297(2.04791)	9.19316(3.07741)	1.44330(1.14073)	2.38837(0.65252)	3.8346(8.787)
-1	E1	7.511411(5.90852)	9.84178(5.95459)	1.92449(2.321159)	2.22641(1.73628)	4.77760(12.489)
	E2	7.54224(5.88524)	9.96573(5.74582)	1.89636(2.30112)	2.18960(1.73830)	4.79590(12.252)
	E3	6.87695(4.74063)	9.29617(5.04415)	1.80805(1.85312)	2.097630(1.35018)	4.46469(12.502)
-2	E1	7.60146(7.34350)	9.82954(6.65917)	2.05578(2.49674)	2.414086(1.81798)	4.78075(14.640)
	E2	7.66106(7.44025)	10.04173(6.52473)	2.04510(2.55870)	2.379686(1.90389)	4.81530(14.279)
	E3	7.03000(6.09232)	9.44932(5.81444)	1.93926(2.06940)	2.267608(1.48778)	4.50226(14.450)

Table 16.1: Summary statistics on M4 when sample size equals 100.

β	0			1			2			-1			-2		
	median			median			median			median			median		
method	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8
M1	2028	2041	1670	2300	2247	1606	2609	2016	1161	1935	1953	1520	1819	1737	1416
M2	2034	2058	1684	2332	2268	1583	2666	1988	1098	1918	1970	1527	1798	1744	1427
M3	2378	1987	1368	2332	2268	1583	2580	1496	653	2240	1876	1331	1974	1751	1249

Table 17: Summary statistics on M4 when sample size equals 200.

β	method	$\overline{m} \times 10^0$ (σ^2 of $m \times 10^0$)	$\overline{\theta} \times 10^0$ (σ^2 of $\theta \times 10^0$)	$\overline{\delta} \times 10^0$ (σ^2 of $\delta \times 10^0$)	$\overline{\varepsilon} \times 10^0$ (σ^2 of $\varepsilon \times 10^0$)	$S(t_i = 7)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.24265(2.13866)	9.79257(2.45708)	1.25505(0.92863)	1.47361(0.66610)	4.77582(5.178)
	E2	7.24170(2.12549)	9.78982(2.37214)	1.23270(0.89813)	1.45578(0.63984)	4.77600(5.142)
	E3	6.59590(1.73077)	9.01686(2.04982)	1.25207(0.79473)	1.46509(0.54697)	4.42423(5.323)
	KM	7.14100(0.74979)	9.75519(0.69794)	0.67127(0.27937)	1.01537(0.16624)	4.78263(1.860)
1	E1	6.94320(1.72754)	9.92361(2.09486)	1.17265(0.84039)	1.58024(0.52290)	4.62588(4.571)
	E2	6.92110(1.68644)	9.80753(2.01811)	1.13788(0.77678)	1.53892(0.48434)	4.61385(4.545)
	E3	6.31410(1.37857)	8.93101(1.70419)	1.19200(0.69934)	1.56365(0.42607)	4.25926(4.682)
2	E1	6.27615(1.29509)	9.76155(1.82558)	1.07619(0.70166)	1.86958(0.42732)	4.26786(4.224)
	E2	6.22445(1.24987)	9.554819(1.77910)	1.06312(0.64921)	1.82179(0.40837)	4.23202(4.220)
	E3	5.70345(1.01838)	8.625263(1.48011)	1.29441(0.70859)	1.91624(0.40728)	3.87876(4.314)
-1	E1	7.31895(2.52814)	9.716326(2.69588)	1.30307(1.01738)	1.52553(0.73940)	4.80611(5.953)
	E2	7.33385(2.52372)	9.78621(2.64300)	1.29262(1.01612)	1.50851(0.74510)	4.81463(5.900)
	E3	6.69055(2.08717)	9.07781(2.33540)	1.30271(0.88647)	1.50556(0.62975)	4.46763(6.096)
-2	E1	7.35290(3.13147)	9.72917(3.25799)	1.43245(1.22929)	1.68350(0.89727)	4.79917(7.235)
	E2	7.38415(3.14496)	9.84635(3.21904)	1.42837(1.25163)	1.66804(0.92649)	4.81563(7.151)
	E3	6.74150(2.61899)	9.17643(2.87592)	1.40708(1.05558)	1.63861(0.75030)	4.47670(7.369)

Table 17.1: Summary statistics on M4 when sample size equals 200.

β	0			1			2			-1			-2		
	median			median			median			median			median		
method	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8
M1	2325	2966	2024	2920	3099	1754	3710	2447	999	2280	2685	2075	2183	2518	1890
M2	2330	2971	2031	2956	3105	1746	3739	2403	924	2251	2705	2081	2152	2512	1916
M3	3106	2670	1463	3637	2441	1051	3568	1490	371	2934	2628	1406	2707	2414	1414

Table 18: Summary statistics on M4 when sample size equals 400.

β	method	$\overline{m} \times 10^0$ (σ^2 of $m \times 10^0$)	$\overline{\theta} \times 10^0$ (σ^2 of $\theta \times 10^0$)	$\overline{\delta} \times 10^0$ (σ^2 of $\delta \times 10^0$)	$\overline{\varepsilon} \times 10^0$ (σ^2 of $\varepsilon \times 10^0$)	$S(t_i = 7)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1	7.15770(0.99518)	9.65121(1.16168)	0.86019(0.42732)	1.02437(0.29759)	4.78095(2.504)
	E2	7.15745(0.99113)	9.65013(1.13876)	0.85156(0.41901)	1.01815(0.29057)	4.78097(2.495)
	E3	6.52350(0.83173)	8.89186(0.99293)	0.99781(0.46541)	1.13363(0.33126)	4.42664(2.594)
	KM	7.10930(0.41099)	9.62930(0.35016)	0.47191(0.13027)	0.71820(0.07450)	4.78232(0.924)
1	E1	6.83995(0.83609)	9.56801(0.99378)	0.79638(0.35955)	1.09255(0.22162)	4.61208(2.271)
	E2	6.82965(0.82778)	9.49806(0.97170)	0.79240(0.34981)	1.08112(0.21567)	4.60576(2.264)
	E3	6.23840(0.68363)	8.69110(0.82834)	1.06432(0.47872)	1.28299(0.31127)	4.25575(2.342)
2	E1	6.20195(0.66355)	9.21863(0.91669)	0.82809(0.35866)	1.38512(0.22637)	4.26320(2.112)
	E2	6.17260(0.65407)	9.09053(0.90030)	0.86873(0.38131)	1.38413(0.23551)	4.24253(2.107)
	E3	5.66290(0.53551)	8.27536(0.73750)	1.36410(0.56859)	1.69362(0.33185)	3.90523(2.151)
-1	E1	7.25015(1.23187)	9.69805(1.35281)	0.93300(0.49710)	1.08707(0.36208)	4.82214(3.039)
	E2	7.25820(1.23365)	9.73397(1.33900)	0.93060(0.49783)	1.08164(0.36436)	4.82626(3.026)
	E3	6.61880(1.02018)	8.99905(1.18563)	1.00517(0.50815)	1.13914(0.37084)	4.47276(3.147)
-2	E1	7.28710(1.53002)	9.73177(1.63204)	1.01773(0.62038)	1.18986(0.45568)	4.82821(3.645)
	E2	7.30340(1.53570)	9.79104(1.62117)	1.01868(0.62956)	1.18637(0.46678)	4.83616(3.625)
	E3	6.65170(1.27051)	9.06990(1.44640)	1.06280(0.57299)	1.21649(0.41379)	4.48445(3.768)

Table 18.1: Summary statistics on M4 when sample size equals 400.

β	0			1			2			-1			-2		
	median			median			median			median			median		
method	6	7	8	6	7	8	6	7	8	6	7	8	6	7	8
M1	2361	4108	2416	4209	3422	1113	4951	2711	499	2168	3698	2594	2217	3446	2390
M2	2362	4114	2414	3342	4181	1656	5006	2612	456	2154	3687	2615	2200	3430	2415
M3	4209	3422	1113	4942	2769	554	4600	1041	71	3672	3496	1313	3472	3195	1406

Table 19: Summary statistics on M3 when sample size equals 100.
(* describe without modifying).

β	method	$\overline{m}\times 10^{-1}$ (σ^2 of $m\times 10^{-2}$)	$\overline{\theta}\times 10^{-1}$ (σ^2 of $\theta\times 10^{-2}$)	$\overline{\delta}\times 10^{-1}$ (σ^2 of $\delta\times 10^{-2}$)	$\overline{\varepsilon}\times 10^{-1}$ (σ^2 of $\varepsilon\times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.39767(6.1882)	10.00884(6.1682)	2.00318(2.4119)	2.24083(1.9157)	7.34566(5.707)
	E1	7.39475(6.1460)	10.00862(6.1275)	1.99638(2.3982)	2.23481(1.9032)	7.34622(5.670)
	E3*	7.29588(5.7294)	9.90656(5.3925)	1.86041(2.0946)	2.11345(1.6295)	7.32605(5.610)
	E3	7.29747(5.7564)	9.90694(5.4257)	1.86632(2.1061)	2.11862(1.6398)	7.32555(5.645)
1	E1*	7.37038(5.2981)	10.75642(5.4412)	2.07275(2.7179)	2.37905(2.1401)	7.36449(4.865)
	E1	7.34265(5.2372)	10.72609(5.3956)	2.05357(2.6762)	2.36396(2.1012)	7.35603(4.854)
	E3*	7.11122(4.4954)	10.19983(4.6544)	1.76426(2.0283)	2.07911(1.5783)	7.30895(4.871)
	E3	7.13646(4.5321)	10.22763(4.6947)	1.77798(2.0596)	2.08893(1.6080)	7.31754(4.883)
2	E1*	7.29860(4.8374)	11.55243(5.1003)	2.43584(3.3705)	2.84006(2.5952)	7.036403(4.785)
	E1	7.23062(4.7441)	11.47476(5.0570)	2.38170(3.2757)	2.80105(2.5085)	7.33573(4.819)
	E3*	6.82418(3.8563)	10.52433(4.4808)	1.82795(2.1841)	2.24443(1.7653)	7.24222(4.933)
	E3	6.89614(3.9394)	10.59798(4.5253)	1.86121(2.2618)	2.26605(1.8326)	7.27192(4.902)
-1	E1*	7.37894(7.1303)	9.68878(6.8467)	2.08568(2.5310)	2.37684(1.9128)	7.32354(7.065)
	E1	7.39186(7.1117)	9.70179(6.8136)	2.08049(2.5246)	2.37150(1.9087)	7.32771(7.005)
	E3*	7.39146(6.8905)	9.86858(6.2911)	2.00204(2.4168)	2.27996(1.8501)	7.32658(6.843)
	E3	7.38225(6.9148)	985736(6.3181)	2.00576(2.4208)	2.28370(1.8528)	7.32267(6.897)
-2	E1*	7.40361(8.4888)	9.63271(7.7173)	2.20550(2.8697)	2.56897(2.1198)	7.30904(8.889)
	E1	7.42037(8.4867)	9.65068(7.6889)	2.20073(2.8673)	2.56393(2.1191)	7.31454(8.805)
	E3*	7.49108(8.4896)	9.99558(7.3010)	2.15424(2.9033)	2.48552(2.2050)	7.33067(8.502)
	E3	7.47546(8.4846)	9.98036(7.3238)	2.15625(2.9027)	2.48811(2.2026)	7.32568(8.573)

Table 20: Summary statistics on M3 when sample size equals 200.
(* describe without modifying).

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.16304(2.5428)	9.70629(2.8203)	1.33916(1.0685)	1.53780(0.8000)	7.37351(2.712)
	E1	7.16227(2.5294)	9.70615(2.8009)	1.33446(1.0616)	1.53373(0.7939)	7.37375(2.695)
	E3*	7.08213(2.3992)	9.61840(2.5722)	1.27555(0.9585)	1.48413(0.7028)	7.35210(2.693)
	E3	7.08451(2.4286)	9.61862(2.5891)	1.27982(0.9646)	1.48780(0.7081)	7.35187(2.709)
1	E1*	7.14665(2.3759)	10.13859(2.6052)	1.35730(1.1678)	1.60793(0.8816)	7.36301(2.435)
	E1	7.11900(2.3443)	10.10926(2.5832)	1.34387(1.1457)	1.59810(0.8614)	7.35424(2.431)
	E3*	6.96693(2.1615)	9.74436(2.3469)	1.21907(0.9193)	1.47714(0.6754)	7.31469(2.452)
	E3	6.99506(2.1977)	9.77174(2.3674)	1.22689(0.9347)	1.48231(0.6892)	7.32357(2.457)
2	E1*	7.14341(2.2184)	10.65247(2.4817)	1.53349(1.4533)	1.88381(1.0909)	7.37078(2.341)
	E1	7.06624(2.1528)	10.57914(2.4594)	1.49197(1.3933)	1.85682(1.0424)	7.34180(2.359)
	E3*	6.78986(1.8595)	9.93032(2.2483)	1.21665(0.9513)	1.58098(0.7338)	7.26861(2.414)
	E3	6.85886(1.8962)	10.00019(2.2717)	1.23640(0.9909)	1.59078(0.7663)	7.29863(2.397)
-1	E1*	7.15464(3.0254)	9.57035(3.1777)	1.41364(1.1838)	1.64358(0.8710)	7.34950(3.280)
	E1	7.16635(3.0191)	9.58367(3.1623)	1.40976(1.1814)	1.63975(0.8693)	7.35335(3.254)
	E3*	7.13531(2.9163)	9.65291(3.0031)	1.37303(1.1405)	1.59496(0.8451)	7.34293(3.230)
	E3	7.12368(2.9228)	9.64073(3.0167)	1.37609(1.1421)	1.59792(0.8461)	7.33917(3.255)
-2	E1*	7.15785(3.5470)	9.54834(3.6197)	1.51208(1.3352)	1.77692(0.9646)	7.35022(4.151)
	E1	7.17311(3.5446)	9.56693(3.6055)	1.50847(1.3340)	1.77300(0.9645)	7.35537(4.115)
	E3*	7.18430(3.5120)	9.73336(3.4783)	1.48325(1.3315)	1.73282(0.9796)	7.35364(4.060)
	E3	7.16894(3.5127)	9.71661(3.4906)	1.48508(1.3309)	1.73504(0.9782)	7.34870(4.093)

Table 21: Summary statistics on M3 when sample size equals 400.
(* describe without modifying).

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.06129(1.1993)	9.59703(1.3966)	0.94527(0.5120)	1.10155(0.3707)	7.37385(1.355)
	E1	7.06071(1.1915)	9.59691(1.3871)	0.94198(5.086)	1.09873(3.678)	7.37397(1.348)
	E3*	6.98876(1.1479)	9.51466(1.3133)	0.91573(0.4749)	1.07790(0.3383)	7.35192(1.353)
	E3	6.98913(1.1548)	9.51479(1.3221)	0.91883(4.780)	1.08055(3.409)	7.35180(1.361)
1	E1*	7.04287(1.1046)	9.79608(1.3060)	0.92978(0.5278)	1.12703(0.3754)	7.37060(1.247)
	E1	7.01494(1.0884)	9.76782(1.2946)	0.92146(5.160)	1.12123(3.650)	7.36175(1.245)
	E3*	6.89593(1.0325)	9.50651(1.2063)	0.87495(0.4407)	1.08097(0.3006)	7.32619(1.258)
	E3	6.92194(1.0484)	9.53340(1.2173)	0.87788(4.475)	1.08287(3.063)	7.33516(1.260)
2	E1*	7.02697(0.9757)	10.11149(1.2334)	0.99506(0.6145)	1.27237(0.4433)	7.36828(1.143)
	E1	6.95377(0.9566)	10.04087(1.2203)	0.96488(5.795)	1.25504(4.170)	7.33896(1.152)
	E3*	6.74798(0.8748)	9.58815(1.1466)	0.85277(0.4267)	1.14348(0.3068)	7.27578(1.178)
	E3	6.82182(0.8975)	9.65617(1.1600)	0.86037(4.434)	1.14499(3.188)	7.30597(1.170)
-1	E1*	7.05313(1.3781)	9.55797(1.6029)	1.00889(0.5881)	1.16629(0.4351)	7.36454(1.627)
	E1	7.06441(1.3735)	9.57132(1.5950)	1.00644(5.867)	1.16379(4.343)	7.36824(1.615)
	E3*	7.02173(1.3463)	9.57626(1.5408)	0.98912(0.5679)	1.14308(0.4206)	7.35344(1.614)
	E3	7.01055(1.3514)	9.56356(1.5481)	0.99136(5.690)	1.14533(4.213)	7.34977(1.627)
-2	E1*	7.06217(1.6641)	9.53824(1.8363)	1.07990(0.6712)	1.25724(0.4890)	7.36802(1.991)
	E1	7.07664(1.6619)	9.55711(1.8289)	1.07749(6.709)	1.25452(4.891)	7.37303(1.974)
	E3*	7.05521(1.6335)	9.61493(1.7824)	1.06419(0.6625)	1.23588(0.4864)	7.36324(1.968)
	E3	7.03994(1.6326)	9.59720(1.7889)	1.06571(6.621)	1.23771(4.857)	7.35831(1.984)

Table 22: Summary statistics on M3 when sample size equals 100 and $P(T > C) = 0.5$.
(* describe without modifying).

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.35211(5.7804)	10.00080(5.9652)	1.96015(2.3713)	2.20472(1.8749)	7.35964(5.447)
	E1	7.35031(5.7534)	10.00045(5.9254)	1.95343(2.3575)	2.19880(1.8623)	7.36017(5.413)
	E3*	7.24700(5.2322)	9.89902(5.2163)	1.82095(2.0576)	2.08106(1.5913)	7.33981(5.363)
	E3	7.25041(5.2770)	9.89947(5.2490)	1.82684(2.0692)	2.08615(1.6017)	7.33934(5.395)
1	E1*	7.25514(5.6894)	11.89057(8.7896)	3.01061(5.4301)	3.41316(4.3476)	7.34549(5.867)
	E1	7.23428(5.6414)	11.85974(8.7318)	2.98658(5.3701)	3.39370(4.2929)	7.33739(5.851)
	E3*	6.96699(4.8730)	11.16419(7.7224)	2.50249(4.2221)	2.91957(3.4513)	7.27441(5.908)
	E3	6.99033(4.9243)	11.19274(7.7780)	2.52142(4.2783)	2.93411(3.5018)	7.28274(5.925)
2	E1*	6.40128(3.3502)	15.08022(10.7902)	5.66265(9.8395)	6.06436(8.193)	7.36342(6.583)
	E1	6.36459(3.3713)	14.99466(10.7720)	5.58210(9.7799)	5.99950(7.9388)	7.33608(6.626)
	E3*	6.09853(3.0592)	13.50963(10.8637)	4.25543(8.8148)	4.76172(7.4678)	7.18729(6.949)
	E3	6.13565(3.0593)	13.59506(10.9060)	4.32584(8.9450)	4.81502(7.5958)	7.21707(6.907)
-1	E1*	7.24080(4.8807)	9.55749(4.8084)	1.75138(1.7438)	1.97228(1.3525)	7.33652(6.020)
	E1	7.25171(4.8707)	9.57067(4.7844)	1.74649(1.7385)	1.96735(1.3484)	7.34059(5.970)
	E3*	7.27413(4.7880)	9.68521(4.5282)	1.69753(1.6799)	1.91361(1.3082)	7.34666(5.829)
	E3	7.26474(4.8010)	9.67348(4.5487)	1.70124(1.6835)	1.91730(1.3110)	7.34284(5.874)
-2	E1*	7.15125(4.8098)	9.50295(4.4997)	1.69664(1.6208)	0.192039(1.2457)	7.32875(6.562)
	E1	7.16497(4.7962)	9.52184(4.4810)	1.69228(1.6174)	1.91540(1.2439)	7.33415(6.504)
	E3*	7.26028(4.7959)	9.70917(4.3316)	1.66243(1.6105)	0.188240(1.2448)	7.35948(6.289)
	E3	7.24664(4.8051)	9.69247(4.3470)	1.66486(1.6112)	1.88537(1.2444)	7.35454(6.340)

Table 23: Summary statistics on M3 when sample size equals 200 and $P(T > C) = 0.5$.

(* describe without modifying).

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.16706(2.5637)	9.72342(2.8507)	1.34885(1.0801)	1.54582(0.8121)	7.36433(2.699)
	E1	7.16592(2.5486)	9.72321(2.8313)	1.34421(1.0731)	1.54178(0.8059)	7.36458(2.683)
	E3*	7.08951(2.4491)	9.63662(2.6032)	1.28422(0.9719)	1.49178(0.7146)	7.34286(2.681)
	E3	7.09124(2.4653)	9.63685(2.6201)	1.28846(0.9780)	1.49543(0.7199)	7.34262(2.697)
1	E1*	7.23378(3.1781)	11.09931(4.2673)	2.06599(2.5498)	2.42211(1.9715)	7.37119(2.861)
	E1	7.20485(3.1314)	11.06821(4.2370)	2.04457(2.5092)	2.40613(1.9356)	7.36258(2.855)
	E3*	6.98104(2.7737)	10.53032(3.8723)	1.72621(1.9495)	2.10249(1.5365)	7.30774(2.900)
	E3	7.00961(2.8133)	10.55928(3.9016)	1.74210(1.9840)	2.11339(1.5662)	7.31656(2.907)
2	E1*	7.04208(3.0419	13.80839(6.6523)	4.36538(6.1396)	4.74452(4.9416)	7.38360(3.276)
	E1	6.99366(3.0461)	13.72074(6.6237)	4.28341(6.0727)	4.68275(4.8590)	7.35527(3.299)
	E3*	6.60786(2.6350)	12.50851(6.8928)	3.23658(5.4555)	3.73158(4.6011)	7.23507(3.456)
	E3	6.66174(2.6428)	12.59454(6.9365)	3.30505(5.5760)	3.77904(4.7112)	7.26540(3.434)
-1	E1*	7.09617(2.2089)	9.54754(2.3554)	1.22633(0.8534)	1.38291(0.6563)	7.35777(2.821)
	E1	7.10756(2.2003)	9.56093(2.3432)	1.22297(0.8509)	1.37944(0.6544)	7.36154(2.799)
	E3*	7.10308(2.1682)	9.60378(2.2645)	1.20206(0.8298)	1.35710(0.6386)	7.35904(2.768)
	E3	7.09300(2.1751)	9.59124(2.2755)	1.20483(0.8317)	1.35996(0.6400)	7.35537(2.789)
-2	E1*	7.05779(2.1979)	9.52552(2.2167)	1.18519(0.8124)	1.34275(0.6281)	7.35552(3.230)
	E1	7.07341(2.1946)	9.54438(2.2076)	1.18241(0.8112)	1.33956(0.6276)	7.36061(3.204)
	E3*	7.11348(2.1936)	9.63465(2.1606)	1.17050(0.8079)	1.32669(0.6261)	7.37102(3.146)
	E3	7.10052(2.1934)	9.61697(2.1687)	1.17218(0.8077)	1.32871(0.6255)	7.36615(3.171)

Table 24: Summary statistics on M3 when sample size equals 400 and $P(T > C) = 0.5$.
(* describe without modifying).

β	method	$\overline{m} \times 10^{-1}$ (σ^2 of $m \times 10^{-2}$)	$\overline{\theta} \times 10^{-1}$ (σ^2 of $\theta \times 10^{-2}$)	$\overline{\delta} \times 10^{-1}$ (σ^2 of $\delta \times 10^{-2}$)	$\overline{\varepsilon} \times 10^{-1}$ (σ^2 of $\varepsilon \times 10^{-2}$)	$S(t_i = 0.3)$ mean $\times 10^{-1}$(var$\times 10^{-3}$)
0	E1*	7.03960(1.1824)	9.58215(1.3858)	0.94104(0.5066)	1.09859(0.3666)	7.36741(1.331)
	E1	7.03877(1.1731)	9.58216(1.3763)	0.93777(0.5032)	1.09577(0.3637)	7.36756(1.324)
	E3*	6.96786(1.1312)	9.50083(1.3023)	0.91227(0.4700)	1.07523(0.3351)	7.34545(1.328)
	E3	6.96828(1.1397)	9.50085(1.3111)	0.91534(0.4732)	1.07787(0.3378)	7.34530(1.336)
1	E1*	7.09350(1.4352)	10.49413(2.1530)	1.39191(1.1995)	1.69515(0.8991)	7.37627(1.448)
	E1	7.06651(1.4135)	10.46374(2.1376)	1.37395(1.1744)	1.68283(0.8782)	7.36748(1.445)
	E3*	6.88973(1.3053)	10.04293(2.0006)	1.17834(0.9045)	1.50473(0.6824)	7.31697(1.471)
	E3	6.91612(1.3213)	10.07151(2.0159)	1.19000(0.9239)	1.51170(0.6981)	7.32598(1.474)
2	E1*	7.21112(2.1619)	12.76914(4.0454)	3.31729(3.7143)	3.66553(2.9935)	7.38422(1.567)
	E1	7.13938(2.1127)	12.68426(4.0222)	3.23882(3.6581)	3.61161(2.9266)	7.35549(1.579)
	E3*	6.73626(1.7280)	11.70948(4.2024)	2.42765(3.1813)	2.90886(2.6936)	7.024907(1.654)
	E3	6.80425(1.7657)	11.79239(4.2351)	2.49133(3.2735)	2.94638(2.7757)	7.27959(1.642)
-1	E1*	7.03771(1.1148)	9.55470(1.2054)	0.87595(0.4408)	0.98667(0.3404)	7.37224(1.422)
	E1	7.04818(1.1105)	9.56809(1.1991)	0.87383(0.4398)	0.98446(0.3397)	7.37588(1.411)
	E3*	7.03151(1.0959)	9.57249(1.1721)	0.86404(0.4304)	0.97406(0.3321)	7.36954(1.405)
	E3	7.02115(1.0991)	9.55956(1.1781)	0.86601(0.4313)	0.97608(0.3328)	7.36594(1.415)
-2	E1*	6.98452(1.0421)	9.50529(1.0747)	0.82802(0.3890)	0.93975(0.2987)	7.36053(1.584)
	E1	7.00074(1.0404)	9.52427(1.0702)	0.82589(0.3885)	0.93738(0.2985)	7.36555(1.571)
	E3*	7.01509(1.0366)	9.56471(1.0549)	0.81996(0.3864)	0.93098(0.2972)	7.36900(1.555)
	E3	6.99979(1.0365)	9.54637(1.0591)	0.82143(0.3862)	0.93267(0.2968)	7.36409(1.568)