# The First-Passage-Time Moments for Hougaard Process and its Birnbaum-Saunders Approximation

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## Abstract

Hougaard processes, which include gamma and inverse Gaussian processes as special cases, as well as the moments of the corresponding first-passage-time (FPT) distributions, are commonly used in many applications. Because the density function of a Hougaard process involves an intractable infinite series, the Birnbaum-Saunders (BS) distribution is often used to approximate its FPT distribution. This article derives the finite moments of FPT distributions based on Hougaard processes and provides a theoretical justification for BS approximation in terms of convergence rates. Further, we show that the first moment of the FPT distribution for a Hougaard process approximated by the BS distribution is larger and provide a sharp upper bound for the difference using an exponential integral. The conditions for convergence coincidentally elucidate the classical convergence results of Hougaard distributions. Some numerical examples are proposed to support the validity and precision of the theoretical results.

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**Keywords:** characteristic function; contour integration; exponential dispersion model; residue; Stirling numbers.